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Unit 1, Example 7: Adding and subtracting negative numbers

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First though I'm going to remind you how to add a positive number. For example, to calculate three plus two - well of course you know how to do that, but let's see how it works on a number line. And here's my number line. I'll put zero there ... one, two, three. So I'm starting at three, and this plus two, well that means I must move two units to the right from three. Two units to the right, and that takes me to four, five. So the answer is three plus two equals five. And of course I would have got the same answer if I'd started at two and moved three units to the right. That would also have brought me to five.

Well let's do another example, minus five plus three equals. And again I'm going to do this using a number line. So here's my number line. The point minus five I'll place here. And the plus three means I must move three units to the right. One, two, three, plus three, and that takes me to minus four, minus three, minus two. So the answer to that is minus two. Now a calculation like this could arise for example when you're thinking about temperatures; the temperature today could be minus five degrees, it goes up by three degrees and it gets to minus two degrees. So a calculation like that could be interpreted in terms of temperatures.

Now I'm going to remind you how to subtract a number. So suppose we want to calculate eight subtract three. You might also hear that said as eight take away three or eight minus three. Well let's do this again on a number line. You know how to do a subtraction like this, but we'll see how it works on the number line. I'm going to put my number eight here, and subtracting three means moving three units to the left, so one, two, three. So that's, I'm going to subtract three and that takes me to eight, seven, six, five. So eight subtract three is five.

Well let's do another example, to calculate minus one subtract four. Again we'll do this on the number line. So I'll put minus one here. And subtract four means I've got to move four units to the left, so one, two, three, four, in order to subtract four, and that takes me to minus two, minus three, minus four, minus five. So the answer is minus five. So minus one subtract four gives me minus five. Well again you can think of this in terms of temperature, if the temperature is minus one degree, and it drops by four degrees, then you get to minus five degrees.

So, summarising this, if we add a positive number it means we move to the right, and if we subtract a positive number it means we move to the left. But what happens if we add or subtract zero? This leaves the number

unchanged so we don't move at all. For example, two plus zero, well that's two. Fifty-three subtract zero is fifty-three. Minus two plus zero and that's minus two.

Well, you can write that last one the other way round as zero plus minus two equals minus two. Let's see how that works on the number line. Well, we're starting with our number zero, and we're adding minus two to it and the answer is minus two. So adding minus two is like subtracting two. So let's write that down; adding minus two is the same as subtracting two. And it works in the same way for any negative number. Adding a negative number is the same as subtracting the corresponding positive number.

So now we've gone through that rule, let's try those two examples you saw at the beginning. Okay, the first one is minus three plus minus six. Well adding a negative number is the same as subtracting a positive number, so this is minus three subtract six, and that's minus nine. Let's look at the other one; four plus minus two. Well adding a negative number is the same as subtracting a positive number. So this is four subtract two, and that's just two.

Now we're going to do some examples of subtracting negative numbers. So I'm going to talk you through subtracting negative numbers. Let's go back to this subtraction we discussed earlier; eight subtract three is five. You can think of this a slightly different way, let's mark eight and three on the number line, and the answer there five tells you what you have to add to three in order to get to eight. So the answer to this subtraction is the number which you have to add to three to get to eight.

Now let's look at another example; three subtract minus four. Well I'll draw a number line, and I'm going to mark three on the number line and minus four. And the answer to this subtraction is going to be the number which I have to add to minus four in order to get three. Well zero is four to the right of that, and it's another three to get to three, so all together this is plus seven. So the answer is three subtract minus four is seven.

Okay, so this is the same sum as three plus four equals seven. So subtracting minus four is the same as adding four. Subtracting minus four is the same as adding four. And, in general, whenever you subtract a negative number, the answer is the same as if you added the corresponding positive number. Well let's use that rule now to do those examples I showed you earlier.

The first one is zero subtract minus six. Well subtracting a negative number is the same as adding the corresponding positive number, so this is zero plus six, and that's six. This one, one subtract minus two. So subtracting a negative number is the same as adding a positive number, so this is one plus two and that's equal to three. And the last one, minus two subtract minus three, well subtracting a negative number is the same as adding a positive number, so this is minus two plus three and that's equal to one.

So I hope that now you can add and subtract negative numbers more confidently. Just remember, adding a negative number is the same as subtracting the corresponding positive number, and subtracting a negative number is the same as adding the corresponding positive number.

Unit 1, Example 8: Multiplying and dividing negative numbers

This clip looks at some calculations involving multiplying and dividing negative numbers.

So what are the rules for this? Well look at this addition; minus three plus minus three. Well that is equal to minus three subtract three which is minus six. And another way of writing the left hand side of this equation is to say that two lots of minus three is minus six. And on the left we have now an example of the rule that a positive number multiplied by a negative number gives a negative number. Now the order of multiplication doesn't matter so this is the same as minus three times two which is going to still be minus six. So the rule for a negative number times a positive number should be that it gives a negative number, and that's the rule; if the signs are different, then the result is negative.

So what's the rule to apply for multiplying two negative numbers together? Well for this let's look at this multiplication table. On the top left of the table, we have these positive numbers which correspond to a positive number multiplied by a positive number. On the top right of the table, we have these negative numbers which correspond to a positive number multiplied by a negative number. And on the bottom left of the table we have these negative numbers which correspond to the negative numbers multiplied by these positive numbers. And what we have left is some blanks in the bottom right hand corner which correspond to a negative number multiplied by a negative number, and we want to find out what these entries should be.

Well let's look at the patterns in the table. If we look across this first row, we're multiplying by three and the numbers, the entries in the table, are decreasing by three each time. So we're going from nine to six to three etc. And on the second row of the table we're multiplying by two, and then we're decreasing by two each time. So let's have a look at this first row where we have blanks. Here we're multiplying by minus one, and we look at the numbers, and they're increasing by one each time. So if we want the pattern to continue, they should increase by one each time. In this row we're multiplying by minus two, we look at the numbers, and they're increasing by two each time, so we want the pattern to continue to increase by two each time.

And the bottom row of the table is multiplying by minus three, and the numbers increase by three each time, so we want the numbers here to increase by three each time. But if the pattern continues, we end up with positive numbers in this bottom right corner, which gives the rule that a negative number multiplied by a negative number should be a positive number. And if this rule applies then all the other rules of arithmetic also apply. So that's the one we should use. And although we haven't mentioned it, the rules for division are the same. If you have been dividing things of opposite sign then the answer is negative. If you're dividing things of the same sign then the answer is positive.

Now let's look at the examples we started with. The first calculation we started with is minus five times six. Well we look at the sign first. We've

got a negative multiplied by a positive so the answer is negative, and we've got five times six which is thirty. In the second sum, we have nine divided by minus three which is nine over minus three. Now look at the signs first. We've got a positive divided by a negative which is different signs so the answer is negative, and we've got nine divided by three which is three.

On this third example minus three multiplied by minus seven, look at the signs first. We've got a negative times a negative so the answer is positive, three times seven which is twenty-one.

The fourth example we have minus seventy divided by minus ten which is equal to minus seventy over minus ten. Look at the signs first. We've got a negative divided by a negative which are the same signs so the answer is positive. You've got seventy divided by ten which is seven.

In the fifth example we have the calculation minus two times three times minus four, and looking at the signs first we have minus times a positive which will give a negative number, and then we have that negative number times another negative number which will give a positive number, so the overall result is going to be positive. And the number is two times three which is six multiplied by four which is twenty-four.

Unit 2, Example 10: Estimating the volume of a log

This clip shows you how to substitute numbers into a formula. And here's the formula here. It's a formula that allows you to estimate the volume of a log of wood. I'll just draw a picture of a log of wood. It allows you to estimate the volume of the log if you know the length of the log and if you know the distance round the middle.

So let's have a look at the formula. You can see that on the left hand side of the formula here you've got V, and you're told that V is the volume of the log in cubic metres. And on the right hand side of the formula you've got LD squared divided by four pi, and we're told that L is the length of the log in metres, so that's L, and D is the distance around the middle again in metres, so that's D. And it's divided by four pi.

Now pi is a mathematical constant. It's about equal to three point one four. But you shouldn't use three point one four, instead you should use the button for pi on your calculator which will give you a more precise value for pi.

Right, well let's look at what we're asked to do. We're asked to use this formula to estimate the volume of a log that measures one point five metres long and ninety-two centimetres around the middle, and we're asked to round our answer to two significant figures. Well now that we've read the question carefully we can go ahead and write down our answer.

So we want to estimate the volume of this particular log of wood, and to do that we're going to need to know the values of L and D so we can substitute them into our formula. Well L is the length of the log. So let's start with that. So the length is one point five metres. L is the length in metres, so L is one point five.

Next we need the value of D. Well the distance round the middle, the distance round the middle we’re told is ninety-two centimetres, but we’re told that D is the distance round the middle in metres so we’re going to need to convert this measurement to metres. Now there’s a hundred centimetres in a metre, so to get the distance round the middle in metres we need to divide ninety-two by a hundred, and that’s going to give us zero point nine two metres. So the distance round the middle is zero point nine two metres, so D equals zero point nine two. And now we’ve got everything that we need to substitute into our formula.

So substituting L equals one point five and D equals zero point nine two into the formula gives. So that’s V equals L is one point five times D squared, D is zero point nine two squared, all over four pi. So that’s four times pi. And remember you’re going to be able to get the value of pi from your calculator.

So the next thing that we need to do is to work out the value of this division using our calculator. If you’ve got a natural display calculator you’ll be able to just type this division into your calculator just as it stands, press your equals sign and you’ll get the answer. If you don’t have a natural display calculator it’s a bit more complicated.

You have to be a bit careful because if you type something like this into your calculator, one point five times nought point nine two squared divided by four times pi then you’ll get the wrong answer. You need to remember to put brackets round the four times pi so that the top is divided by the whole of the bottom and not just by the four. You can find more information about using your calculator to work out divisions like this in the IT guide. Well if you use your calculator to work out this division, you’ll find that the answer is zero point one zero one zero and some more figures.

The question asks us to round our answer to two significant figures, so let’s do that. Well, looking from the left, the first significant figure is this one here, the next significant figure, the second significant figure is this zero, and the third significant figure is this one and so on. So we want two significant figures. So we look at the next significant figure along after the two significant figures, and it’s this one here. Now one is less than five so that means that we’re going to round our answer down when we round to two significant figures, and that’s going to give us zero point one zero. To show that we’ve rounded to two significant figures, we write to two s dot f dot in brackets.

So we’ve now finished the calculation, but we have to finish off by writing down a conclusion. So the volume of the log, because that’s what we found, is zero point one zero, and we must remember to put the units in, and the units are cubic metres as we’re told up here in the question, and we write to two s f for significant figures to show that we’ve rounded our answer.

Unit 2, Example 13: Finding a formula - the car ferry

In this clip we’re going to look at this problem which involved finding a formula for the space required for cars and vans on a car ferry. The hardest

part in these sorts of problems is often where to start, and as a general rule a place to start is to try an example first.

So let's look at a particular example. We can start with just a couple of cars. So that's one car, and that requires a space of five metres. And then put another car in. And that's another five metres required. And then let's put a van in. And that requires a space of nine metres. So in this example the space required is… we've got two lots of five metres plus nine metres and that gives to two fives are ten plus nine is nineteen metres for the space required in this particular example. And this should help you lead in for the general case which we'll now proceed to do.

So we start off with looking at the cars. So we've got one car here which requires five metres, and then we have a number of other cars which also require five metres each, and in total we would have C cars - that was given in the question. And what's the space required for the C cars? Well for one car it's five metres, for two cars as we saw before it's two times five metres, so for C cars we have C times five metres which we usually write as five C.

Now let's consider some vans. So let's add some vans to our diagram. And when drawing these diagrams, it's, you don't have to get a good likeness or anything, it's just as your aid memory, as long as it, you know, just vaguely resembles. It's not an art competition. So here we have some vans. Each one requires nine metres in space. We have V vans. And what is the space required for the vans? Well for one van we have nine metres, for two vans we'll have two lots of nine metres, so for V vans we'll have V lots of nine metres, which we usually write as nine V.

So this is the space for the cars and vans separately. So what's the space for both cars and vans? Is, well, this is the variable L that's in the question, and we just have to add up the space separately required for cars and vans. So it's five C for the cars plus nine V for the vans. This is the formula that the question requires. But it's good practice to do a check and to see whether the formula is correct, and for this you can use the example that you've started off thinking about to be getting to the problem.

So let's look at that as our check. In this example we had two cars which corresponds to C equals two, and we have one van which corresponds to V equals one. So let's substitute C equals two and V equals 1 into the formula to see what it gives. So that gives L equals five times C, and C is two, plus nine times V, nine times V, and V is one. So this is five times two is ten plus nine is nineteen. And that is the same as we got before, so that is as expected. And that completes the check. So we have checked that our formula gives the right answer for our example.

Unit 2, Example 15: Using a double inequality

In this clip, we're going to write down some inequalities which can be used to describe child fares on the train. The question tells us that the child fare is available for children five years or older but not yet sixteen, and we've got illustrate on a number line these restrictions. So let's start by drawing a number line.

And the question tells us that we are to use the variable A to represent the age of a child in years, so let’s label our number line with an A. I’m going to mark on the number line a range of numbers that's sufficiently wide for the problem. So I’ll start with four, say, and I’m going to mark even numbers going up, four, six, eight, ten, twelve, fourteen, sixteen, well eighteen, that will be enough.

And the question tells us that these child fares are available for children who are five years or older but not yet sixteen. So I’m going to draw a little circle above the number five and a little circle above the number sixteen. And they’re available for all ages between five and sixteen, and this includes five but not sixteen. So I’m going to fill in this circle, make it into a solid circle to show that value five is included, but I’ll leave this circle empty to show that the number sixteen is not included. So what I’ve done is to draw an interval on this number line which shows the range of ages for which the child fare is available.

The second part of the question asks me to write down a double inequality that describes these restrictions. Well, what I know is that A can be any number between five, including five, and sixteen, but not including sixteen. From the diagram, I know that, from the diagram, I know that A is greater than or equal to five, and I write that down using this symbol, A is greater than or equal to five. So this symbol which is an arrow pointing in the right direction with a little line underneath to indicate that it is a bit like an equal sign is greater than or equals. And if you want to remember which way around it goes, then the number on the smaller side of the arrow is the smaller number. So here A is larger than or equal to five. Well, I know that A is greater than or equal to five, and I know that A is less than sixteen, so A is less than sixteen. And the smaller side of the arrow is on the same side as the smaller number, A is smaller than sixteen.

Well, the question asked me to find a double inequality to describe the age restrictions, so what I’m going to do is to combine these two inequalities together. And the first thing I have to do is to take this inequality and rewrite it in a different form. So I’m going to rewrite it with A and five interchanged, and then I’ll have to make the inequality between go in the opposite direction. So what I get is that A greater than or equal to five is the same as, well I swap the five and the A, and I make the inequality go in the opposite direction, so five is less than or equal to A. So those two inequalities mean exactly the same thing.

Well now I can combine these two inequalities, five less than or equal to A and A is less than sixteen, to make one double inequality. So combining inequalities, I get five is less than or equal to A, that’s this inequality, and A is less than sixteen. So that’s a double inequality to describe the age restriction for child fares.

Well, the last part of the question asks us to say which whole numbers satisfy this inequality, and what we’re going to do is to write down a list of those whole numbers. So the whole numbers that satisfy this double inequality are, well, the number five satisfies it, and then I’ll write a list increasing by one each time, five, six, seven satisfies it, eight satisfies, nine, all these numbers satisfy it, and fifteen satisfies, but sixteen does not satisfy it. So that’s my list of numbers that satisfy this inequality.

So in this question you saw how to describe a real life situation using inequalities. First we described the situation using two inequalities, A greater than or equal to five and A less than sixteen, and then we combined those inequalities to produce one double inequality.

Unit 3, Example 4: Adding and subtracting fractions

This clip shows you how to add and subtract fractions. Let's look at the first example here; three eighths plus one eighth. Well in this case the two fractions that we're asked to add have got the same denominator, and adding fractions is easy if you've got the same denominator, you just add the numerators. So the denominator of the answer is going to be eight, we add the numerators, three plus one, and so we get the answer four over eight.

Now when you get an answer that's a fraction you should always make sure that you express it in lowest terms. If you look at this fraction we can see that it's not in lowest terms because both the numerator and the denominator are divisible by four. So let's cancel that four. Well four goes into four once, and it goes into eight twice, so the answer is a half.

Let's look at the next example. We're asked to add two fractions again, but in this case we can see that the denominators are not the same. So let's begin by writing down the question; four over nine plus five over six. Before we can add the fractions, we're going to have to express them with the same denominator, so we need to write this fraction here as an equivalent fraction and we need to write this fraction here as an equivalent fraction, such that the two fractions have the same denominator.

So to write this fraction as an equivalent fraction we've got to multiply the top and the bottom by the same number, and the same for this fraction over here. So the first thing that we need to do is think about what number we could have for our new denominator, and we need to find a number that's exactly divisible by both nine and six. Well a little bit of thought should show you that a good number to use is eighteen, because eighteen is divisible by nine and also divisible by six.

So we're going to write both our fractions with the denominator eighteen. So we need to multiply top and bottom of this fraction by the same number so that we end up with eighteen on the bottom. Well you need to multiply nine by two to get eighteen, and so you need to multiply four by two to get an equivalent fraction and that will give you eight. Doing the same thing you need to multiply six by three to get eighteen, so you need to multiply five by three so that you get an equivalent fraction, and that will give you fifteen on the top. So now we're back to the situation that we had in Part A, we've got the same denominator, so we just go ahead and add the numerators. Eighteen will be the denominator, and on the numerator we're going to have eight plus fifteen which will be twenty-three.

We look to see whether any cancellation can be done, whether this fraction is expressed in lowest terms, and in this case we see that we can't do any cancellation, it is in lowest terms. The only other thing that you might notice here is that this fraction is top heavy; the numerator is bigger than the denominator. There's nothing wrong with that, you can just leave your

answer as a top heavy fraction, but you might like to convert it into a mixed number so that you can get a better idea of just how big this number is.

Well to convert it into a mixed number you need to look at how many times the denominator goes into the numerator. In this case it goes in once. So the integer part of the mixed number will be one. And when you divide eighteen into twenty-three it goes once, as we said, and the remainder is five, so the fractional part will be five over eighteen.

The trickiest part of this example was finding the number that we wanted to be the denominator of the two fractions. We chose eighteen. If you're not sure what number to pick, you can always just multiply together the denominators of the two fractions. In this case you would have multiplied nine and six to get fifty-four. The calculation would have gone through just as before, and at this stage you would have ended up with a fraction which had fifty-four as the denominator, and you would have had to have cancelled it down to get twenty-three over eighteen.

Let's now look at Part C. In this part we're asked to subtract two fractions. Subtracting fractions works in just the same way as adding them. If you've got the same denominators then you just go ahead and subtract the numerators. If the denominators are different, then first of all you have to express each fraction as an equivalent fraction in such a way that you've got the same denominators.

So let's go ahead and do this one. Six over seven minus two thirds. Well we need a number that's a multiple of both seven and three, and in this case the best number to use is just the one that you get by multiplying together the two denominators and that's twenty-one. So we're going to express the two fractions with denominator twenty-one. Well, to get twenty-one from seven you need to multiply by three. So we're going to need to multiply the top by three as well, and that will give 18. To turn three into twenty-one you need to multiply by seven, so you've got to multiply the top by seven as well, and that will give fourteen. And now we just subtract the numerators, so that will give us four over the denominator twenty-one and that's our answer. To finish off we just make sure that our fraction is expressed in lowest terms, and in this case it is, there's no cancelling to be done.

In the final example we're asked to add together two mixed numbers, one and two thirds and four and a half. So one and two thirds plus four and a half. Well to add mixed numbers you can add the integer parts and you can add the fractional parts and see what you end up with. So if we add the one and the four we get five, and then we're going to add the two thirds and the half using the method that we used above.

Well remembering that the fractional parts are always added onto the integer parts, we're going to put a plus here so that we don't forget that, and then we're going to have two thirds plus a half. Okay, let's simplify that where we're going to have the five, still from here, and we're going to want to add these fractions. So we're going to have to express them with a common denominator, we're going to have to find a number that both three and two divide into exactly, and that number will be six.

So we're going to express both the fractions with the denominator six. Right, well we have to multiply three by two to get six, so we'll have to multiply the numerator by two as well and that will give four. For this fraction we have to multiply the two by three to get six, so we have to multiply the one by three as well giving us three. So now we can add the fractions by just adding the numerators. So we carry forward the five, it's still there. And the fractional part will have the denominator six, and the numerator is four plus three which is 7.

Now we're going to have to express this as a mixed number, and this part here is a top heavy fraction so we're going to have to express just this part alone as a mixed number so that we can add the integer part of this top heavy fraction onto the five. So the next stage is to convert this top heavy fraction into a mixed number. So again we carry forward the five because it's still there, and we've got seven over six. Well six goes into seven one time and it's got remainder one, so it's one and one sixth. And now we can just add together this integer here and the integer part of this mixed number to obtain six and one sixth.

Unit 3, Example 5: Multiplying fractions

In this clip I'm going to show you how to multiply fractions. Well to multiply fractions you just multiply the numerators and multiply the denominators.

Look at the first example. It's two fifths times four sevenths. So if we multiply the numerators we get two times four, and if we multiply the denominators we get five times seven. Two times four is eight, five times seven is thirty-five, so that's finished.

The next example is two times three sevenths. Well two is not a fraction and you can do this by saying that you've got two lots of three sevenths, so the answer is six sevenths. But you could also do it by using the rule for multiplying fractions, because you can write two as two over one, so this is two over one times three over seven. And now if you multiply the numerators we get two times three, the denominators one times seven, and that's six over seven again.

Now let's do two thirds times five sixths, two thirds times five sixths. Well if we multiply the numerators we get two times five, the denominators three times six. And now before we multiply those out, notice that two and six have got a common factor of two. Two divides into both of them, so let's cancel that. We cancel the two there, cancel the two there to give three, and that's one times five over three times three, that's five over nine.

In this last part we have to multiply a fraction five over six by a mixed number three and two thirds. Well when you multiply by a mixed number you always have to convert it into a fraction first, so let's do that. Three times three is nine plus two is eleven, so we have eleven divided by three, and that's multiplied by five sixths.

Well we do this in the usual way; eleven times fives is fifty-five and three times six is eighteen, and that's the answer. But if you wanted to give your answer in the form of a mixed number, then you'd have to convert it into

that form. Well eighteen divides into fifty-five three times, three eighteens are fifty-four, and the remainder is one, so the answer would be three and one eighteenth.

Unit 3, Example 6: Dividing fractions

In this clip I'm going to show you how to divide fractions. If you have one fraction divided by another fraction then you find the answer by turning the second fraction upside down to give its reciprocal and then multiplying.

Look at the first example. It's four sevenths divided by five sixths. So following the rule I just described, we write down four sevenths and then we take the second fraction five sixths, turn it upside down and multiply it. And we do the multiplication in the usual way. Four times six is twenty-four, seven times five is thirty-five, and that's the answer. And it doesn't simplify any further because there are no common factors in the numerator and the denominator.

Now look at this example. Five over six divided by a quarter. Well proceeding in the way that we did before this is five over six times what we get by turning this second fraction upside down, times four over one. And now before multiplying out the numerators and denominators I'm going to notice that the four and the six have a common factor of two. And you can see that if you multiply together the numerators and the denominators then you'd have a factor of two on the top and a factor of two on the bottom which you'd be able to cancel, so let's do that. I'm cancelling the two from four to leave two, and I'm cancelling the two from six to leave three, and then my answer is five times two, which is ten, divided by three times one which is three. So that's my answer ten over three. Well ten over three is a top heavy fraction so I could if I wanted turn it into a mixed number by dividing out by the three, but I won't do that here.

Look at the final question. Three over five divided by two. Well two is not a fraction, but I can write it as a fraction by writing three over five, three fifths, divided by two over one. And now I can use the rule for dividing by fractions to get three over five, three fifths, times one over two, and then that's equal to three times one is three, divided by five times two is ten, and that's my answer, and there is no further cancellation that can take place there.

Unit 3, Example 8: Converting numbers to scientific notation

In this clip we're writing numbers in scientific notation, and the first example is five hundred and twenty-three point four to write in scientific notation.

Well to write the numbers in scientific notation we need a number between one and ten multiplied by a power of ten. In this case the number between one and ten is obtained by moving the decimal point two places to the left, so we have the number five point two three four, and this has to be multiplied by a power of ten to make this equation true. Here we've moved

the decimal point twice to the left, and each time we've made the number ten times smaller. So to keep the equation true, we need to multiply twice by ten, that's ten squared.

Now a frequent mistake in these is to get the wrong order of magnitude for the answer. So here we can do a quick check to see that we have the right order of magnitude. The number on the left hand side of the equation is around about five hundred, and on the right hand side of the equation we have a number that's around five multiplied by ten squared which is a hundred so this is five hundred as well, so this is the right order of magnitude.

The second example is zero point zero zero six seven one to write in scientific notation. To get the number between one and ten we have to move the decimal point three places to the right. So we get a number six point seven one, and we know this has to be multiplied by a power of ten to make this equation true. We have moved the decimal point three places to the right in this case, and each time it corresponds to multiplying this number by ten. So to keep it true we have to divide by ten each time, so we get ten to the minus three here. If you move the decimal point to the right, then you get minus three.

Now again we can check that this is the right order of magnitude. If the six was here in the first place after the decimal point, this would be six tenths. If it's here in the second place, then it would be six hundredths, but it's here in the third place so this is six thousandths. And on the right hand side we have a number which is about six multiplied ten to the minus three which is a thousandth. So on the right hand side we have six thousandths which is the right order of magnitude.

Unit 3, Example 10: Simplifying square roots

In this clip we're going to look at simplifying surds, and the first example is the square root of eighteen. Now you can write eighteen as nine times two, and there's a general rule of surds that the root of a product is the product of the roots. So we can write this as the square root of nine times the square root of two. And the square root of nine is three, so we can write this as three root two, and we considered this to be a simplified form. I will give you some general comments after I've done the next example.

So here's the next example, square root of ten, and in ten you can do exactly the same thing and write that as a product of five times two. Now you use the same general rule of roots, that the root of a product is the product of the roots to write that as five times the square root of two. But when we get to this point something is different, because now neither of these simplifies to be an integer, and this is because this five is not a square number whereas the nine was a square.

So this is not a simplified form, and we would say that the simplified form was indeed just the square root ten that we started with. And this gives us our general principle for determining when a surd in its simplest form. If the number underneath the square root sign has no factors which are square numbers, except one of course, then the surd is in its simplest form.

Let's look at another example. The square root of sixty, how can we factorise this? Well sixty is even and half of it is thirty is also even, so we can take out a factor of four, and it's four fifteens which are sixty. And we do the same trick as above and write this root of a product as the product of roots. And then the square root of four will simplify to be two, so that's two root fifteen. And this is in its simplest form because the number under the square root, fifteen, does not have any factors which are squares, just has three and five.

So let's turn to the last example, the square root of eighty. Now eighty has a factor of four, so it's four twenties, so we can take this out, and the preceding bit as before, we can write this as a square root of four times the square root of twenty. Square root of four simplifies to two, two square root twenty. But we aren't finished at this point because twenty has a square that is a factor. In fact we can write twenty as four times five, so this square root twenty is a square root of four times the square root of five, and the square root of four is two again which multiplies by this other two to give the answer as four square root five.

Now you might have spotted that sixteen was a factor of eighty and done this in one step to take out the square root of sixteen to get four, but if you didn't spot that then you can always do the same method that I have done here.

Unit 3, Example 11: Multiplying roots

In this clip we're going to multiply surds. Here's our first example. The square root of three times the square root of three. Well the square root of three has the property that the square root of three squared is equal to three. So the square root of three times the square root of three is three.

Now you could do that another way by using our rule that the root of a product is the product of the roots, and if you do it this way you get this. The square root of three times the square root of three is equal to the square root of three times three, which is the square root of nine and of course that's three, so you get the same answer both ways.

Let's look at the second example. Two root five times four root five. Well all these numbers, two, the square root of five, four and the square root of five, they're all multiplied together. So we can begin by multiplying together the two and the four to give us eight times the square root of five times the square root of five. Well just as in Part A we've got a product here of two equal square roots, and the square root of five times the square root of five is equal to five, so this is eight times five and that's equal to forty - so that one's done.

Part C, the square root of six times the square root of three. Well now we're going to use our rule that the root of a product is the product of the roots to write this in the form of the square root of six times three. Six times three is eighteen. So there's an answer. But this surd is not in its simplest form, and if we want to put it in its simplest form we notice that eighteen is nine times two. Now nine is a square number, nine is three squared, so we can

write this as the square root of nine times two which is three times the square root of two, so there's an answer in its simplest form.

Let's look at this final example. Five root two times three root ten. Well again this is a product of four numbers, five, root two, three and root 10, so we can begin by multiplying the five and the three to get fifteen times the square root of two times the square root of ten, and that's fifteen times the square root of two times ten, which is fifteen root twenty. Now that's an answer, but again it's not in its simplest form because twenty has a factor of four which is a square number. So this is fifteen the square root of four times five, the square root of four is two, so this is fifteen times two times root five. Well fifteen times two is thirty, so our final answer is thirty root five.

Unit 3, Example 12: Dividing roots

In this clip we're going to divide surds. Let's look at the first example. The square root of fifteen divided by the square root of three.

So what we're going to aim to do is to simplify this just like any fraction by trying to find common factors in the numerator and the denominator. Now we're going to use our rule for the square roots of products, the square root of fifteen is the square root of three times five divided by the square root of three, and then the numerator could be written as the square root of three times the square root of five divided by the square root of three. And now you see we can cancel the square root of three here with the square root of three there, and that leaves our answer as the square root of five.

Now look at this example. Two divided by the square root of two. Again what we aim to do is to cancel common factors in the numerator and the denominator. We've got a square root of two in the denominator, but we have no square root of two in the numerator, but we can arrange to have a square root of two there by writing two as a product, the square root of two times the square root of two. And now you see we can cancel the square root of two here with the square root of two there, and that leaves our answer as the square root of two.

Unit 3, Example 13: Adding and subtracting roots

This clip is about adding and subtracting surds. Let's look at the first example. Well it's two root three plus four root three. So we've got two lots of root three plus four lots of root three, and adding them is really quite simple, it just means we've got six lots of root three. Two lots of root three plus four lots of root three is six lots of root three.

Let's look at the next example. Well this one is about subtraction. So we've got five root two minus root two. And it's very similar to the example above, we've got five lots of root two minus one lot of root two, so that means you'll have four lots of root two.

Let's look at Part C. So here we have root three plus two root five. Now this example is different from Parts A and B because in this case the numbers under the root signs are different. We've got a three and a five,

whereas here we had a two and a two and a three and a three. Now because we've got different numbers under the root signs, there's nothing that we can do to simplify this sum. One lot of root three plus two lots of root five there's nothing we can do with it, so this can't be simplified.

Now look at Part D, that's root twelve minus root three. Now you might think that this part looks similar to Part C because the numbers under the root signs are different. But there's something else to notice about this part, and that's that root twelve is not in its simplest form because twelve has the factor four and that's a square. So let's begin by writing root twelve in its simplest form.

Well as we said twelve is a factor four, so it's four times three, and we've still got the minus root three. Root four times three is the same as root four times root three, and we carry forward the root three. And root four is two, so express the root twelve as two root three and carry forward the minus root three. And let's look at what we've got now.

Well now we can see that we've got something much more similar to Parts A and B, we've got the numbers under the root signs being the same. In fact we've got two lots of root three minus one lot of root three and that will give us one lot of root three, and that's our answer.

Unit 5, Example 9: Identifying terms

When you're asked to simplify an expression, like the expressions here, you can do it by simplifying each term individually.

So the first step is to identify the terms to make sure that you know which bits of the expression belong to which term. So let's do that with the first expression here.

Well, it's minus two A, minus minus five A squared, plus minus four A.

You can identify the terms by using the rule that each term after the first starts with a plus or minus sign that isn't inside brackets.

So the first term starts here, and we go along until we meet a plus or minus sign that isn't inside brackets, and that's here. This minus sign is the start of the next term.

So the next term starts here. We keep going along until we meet a plus or minus sign that isn't inside brackets, and that's here. The plus sign is the start of the next term.

So it starts here, we go along until we meet a plus or minus sign that isn't inside brackets, and this time we don't meet one; we just get to the end of the expression.

So this expression has three terms: minus two A, minus minus five A squared and plus minus four A.

Let's look at the second expression. It's two X times four XY minus two Y times minus five X.

Now, one thing to notice about this expression is that it contains an X. It's got the letter X. When you write the letter X as part of an expression, you

should make sure that it doesn't look like a multiplication sign. I've done that by writing a curly X.

Let's now identify the terms of this expression. Well, again, we use the rule that every term after the first starts with a plus or a minus sign that isn't inside brackets.

Here's the start of the first term. We go along until we meet a plus or minus sign that isn't inside brackets, and that's here; this minus sign. That's the start of the next term.

So the next term starts here. We go along until we meet a plus or a minus sign that isn't inside brackets, and we don't meet one; we just get to the end of the expression. This minus sign here was inside brackets.

So this expression has two terms. The first one is two X times four XY and the second one is minus two Y times minus five X.

Unit 5, Example 10: Simplifying expressions with more than one term

Here we've been given two expressions to simplify, each made up of several terms which are added or subtracted.

Now let's look at the first expression: minus two A minus minus five A squared plus minus four A.

Okay, in order to simplify it, we first have to identify the terms in this expression, and we do this by starting at the left and going along until we reach the first plus or minus sign that's not in brackets, so that's there, and then repeating that. So that's the second term and there, at that plus, that's the third term.

Okay, so now we're going to simplify each of these terms. Well, the first term, minus two A, there's nothing we can do to that, so let's just write it down. But the second term is more complicated; it's minus minus five A squared. Well, here we're subtracting a negative, and that's the same as adding, so that gives us plus five A squared. And then, in this third term, well, here we're adding a negative, and that's the same as subtracting, so that gives us minus four A.

Okay, so we've sorted out the three terms, but now we must check to see whether there are any like terms that we can combine. Well, this is a term, minus two A is a term in A. Minus four A is also a term in A. So we can combine those to make minus six A. And then the plus five A squared, that's by itself, so we write that down. And that's our first expression simplified.

Now, let's look at the second expression: two X times four XY minus two Y times minus five X. Once again we've got to identify the terms, so let's start on the left and go along until we reach the first plus or minus sign that's not in brackets, that's there, and we start at that minus sign and go along again.

So we have two terms. Let's multiply out the first term where we have two X times four XY, the coefficients two times four give us eight, and then X

times XY gives us X times X, which is X squared, times Y. So that's the first term simplified.

But the second term, we've got minus two Y times minus five X. Well, two negatives make a positive, so that's plus. Two times five is ten. And Y times X, well that's YX, but I'm going to write it as XY because it's usual to write these in alphabetical order.

Now I have to check whether there are any like terms, but the term here is XY, the term here is X squared Y, and they're not like terms, so I've finished.

Unit 5, Example 11: Multiplying out brackets

This example is to multiply out the brackets in this expression; the expression two A into three A plus two B. And when multiplying out brackets in this way, it's important to multiply by the multiplier two A by each term in the brackets. In this case three A is the first term and plus two B is the second term.

So let's write that out. The multiplier is two A, and it has to be multiplied by the three A. And for the second term we have the same multiplier, two A, and the second term is plus two B, so that's times two B. And now let's simplify each term. For the first term, we have two times three is six, A times A is A squared, and for the second term we have two times two is four, A times B is AB.

Now when we get a little bit more practiced at doing this, we'll omit this intermediate step and go directly from the first step to the final step, and the way you do this is to do the multiplication in your head, so two times three is six, A times A is A squared, plus two A and two B, and the second term, and two times two is four, A times B is AB.

Unit 5, Example 12: Multiplying out brackets involving minus signs

In this example, we're asked to multiply out the brackets in an expression. Let's begin by writing down the expression. It's minus A times B minus A plus seven.

Well, to multiply out the brackets in an expression like this, what we need to do is multiply the multiplier, which is minus A, by each term inside the brackets. And there are three terms inside these brackets; there's B, minus A and plus 7. So let's multiply it out.

Well, the first term in our answer is going to be minus A times B. So that's a negative times a positive, which will give us a negative, and then we've got A times B, which is AB.

Now we multiply the minus A by the second term, which is minus A again. So this is a negative times a negative, which will give us a positive, and we've got A times A, which gives us A squared.

Finally, we multiply the minus A by the third term in the brackets, which is plus seven. Well, that's a negative times a positive, which will give us a negative, and we've got A times seven, which is seven A, and that's our answer.

Unit 5, Example 13: Expanding the brackets when there's more than one term

So here we've been given two expressions, and in each of them we've got to multiply out the brackets and simplify wherever possible. Let's look at the first expression, X times Y plus one, plus two Y times Y plus three. So what we're going to do is to identify the terms and multiply out these brackets.

So, the usual way, we start on the left and go along until we reach the first plus or minus sign not in brackets. That gives us one term and there's the second term.

So we're going to multiply out the brackets in each of these. So here we've got to multiply the multiplier which is X by each of the terms inside the brackets. So we've got to multiply the X by the Y and the X by the one and write those down. So X times Y is XY and X times plus one is plus X.

Now, let's look at the second term. Well, this is plus two Y times a bracket which has two terms in it, Y and plus three. So let's write down the result of plus two Y times Y. Well, that's plus two Y, Y times Y is Y squared. And then plus two Y times plus three, and that's plus six Y.

Now we check to see whether there are any like terms. This is an XY. This is X. This is Y squared. This is Y. All four are different, so there are no like terms.

Now look at this expression. Two R squared, minus R, times R minus S. We have to simplify this. So let's identify the terms. The first term is two R squared and the second term is minus R times R minus S.

Well, the first term, two R squared, well, you can't simplify that any further so let's write that down, two R squared. But now the second term, minus R times R minus S, we've got a bracket to multiply out. The multiplier is minus R, and that has to be multiplied by each of the terms inside the brackets, R and minus S.

So minus R times R, well a negative times a positive makes a negative, so that's minus, R times R is R squared. Now we have to multiply minus R times minus S, where the negative times a negative makes a positive, so that's plus R times S is RS.

Next, we have to decide whether we can combine any like terms. Well, this is a term in R squared, and this is a term in R squared, so we can combine them. Two R squared minus R squared is R squared, and then plus RS. So we're finished.

Unit 5, Example 14: Plus and minus signs in front of brackets

This example is to remove the brackets from these expressions. The first expression is minus, minus P squared plus two Q minus three R. And the rule is that if there's a minus sign outside the brackets, then we change the sign of each term inside. So, in this case, there is a minus sign.

So the first term is plus P squared. The second term begins with a plus so it's minus two Q. The third term is a minus so that becomes plus three R. And this simplifies further because by convention we usually omit the first sign if it's a plus, so this is P squared minus two Q plus three R, and that's the answer.

The second expression is A plus two BC minus D, and in this case the brackets has a plus sign outside of it, so that means we leave the signs as they are. So this becomes A plus two BC minus D, and that is the simplified expression.

Unit 5, Example 17: Proving a property of numbers

In this example, we're asked to prove that the sum of any three consecutive integers is divisible by three. So the idea here is that it's saying that if you pick any three consecutive integers, like two, three and four, or six, seven and eight, and you add the three integers together, then you always get a number that's divisible by three; in other words, a multiple of three.

Now, you could pick lots of different groups of three integers, but no matter how many you pick and test to see whether they're divisible by three, when you add them up, you can't prove it happens for all of them because there's infinitely many to check. But you can prove that it happens for all of them by using algebra. And here's how you do it.

Well, you represent the first integer by N. So represent the first integer by N. And this is going to work because N will represent any integer. So we're going to prove it for all integers all at the same time by using algebra.

Well, if the first integer is N, then we can say what the other two integers are in terms of N. So the other two integers are. Well, the next integer after N will be N plus one, and the one after that will be N plus two. So our three integers are N, N plus one, N plus two.

Well, what we want to know is if you add them do you get something that's divisible by three. So let's add them. So then the sum of the integers is, well, it's going to be N plus N plus one plus N plus two.

Now, these brackets aren't needed here. We can just take them out because all we've really got here is five things all added up together; N, another N, one, another N and two. So that's just the same as N plus N plus one plus N plus two. And now we can simplify this expression by collecting like terms. Well, we've got N plus another N plus another N, so that's three N, and we've got one plus two so that's plus three.

Now, what we wanted to know is when we add our three numbers together, do we get something that's divisible by three, and you can see that, indeed,

we have got something that's divisible by three because three N must be a multiple of three, and if you add three to a multiple of three, well, you're going to get another multiple of three. So, indeed, three N plus three are a multiple of three.

Now, sometimes if you do something like this, you might find that the expression you get here is a bit more complicated, and it's not quite so obvious whether it's a multiple of three or whatever you're interested in, and you can give a more formal proof in that case. So let's just go through how you would do that.

Well, to find out whether something is divisible by three, what you need to do is try dividing it by three and see whether what you get is an integer. So let's do that. Let's see what happens if you decide by three.

So dividing by three gives, well, that's going to be three N plus three divided by three. We can expand this fraction, which means that you take each term on the numerator and you divide it individually by the term and the denominator, by the denominator. So that's three N divided by three plus three divided by three. Well, three N divided by three, that's just going to be N, because if you take N, and you multiply by three and then you divide by three, you're just going to get N, and three divided by three is just one.

So we wanted to know if you take three N plus three, and you divide by three, do you get an integer, and indeed you do. N plus one must be an integer because N we know is an integer and so N plus one is an integer.

So we did, indeed, get an integer. Let's write that down. So N plus one is an integer, and we know that because N is. So dividing the sum of the three numbers, dividing the sum of the three numbers by three has given us an integer. In other words, the sum is divisible by three.

So that proves the thing that we were asked to prove. If you add any three consecutive integers, then the result is always divisible by three, no matter which three consecutive integers you choose.

Unit 5, Example 21: Solving a more complicated equation

Here we're asked to solve the equation five X equals three X plus ten. So that's an equation involving the unknown X, and we've got to find the value of X. And the strategy we're going to use is to apply steps to both sides of the equation, doing the same thing each time, to try and turn the equation into a simpler form so we can find the value of X. And the first thing we're going to do is try and express the equation in the form a number times X; some number times X is equal to another number. So the first stage of our strategy is to try and express the equation in that form.

Now, when you look at the equation, you can see that, well, it's a number times X on the left, which is what we want, but on the right we've got a number times X as well, this three X. So our problem at the moment is dealing with this term, three X. But we can get rid of that term if we subtract three X from both sides of the equation. So let's do that.

Subtract three X. So on this side we get five X minus three X and on the other side of the equation it's three X plus ten minus three X. So we've subtracted three X from both sides, let's simplify that. Well, five X minus three X is two X. Three X plus 10 plus minus three X, well, three X minus three X, they're going to cancel and leave nothing, so on the right hand side we only have ten.

So we've succeeded in turning our equation by doing this operation of subtracting three X into the form a number times X equals a number. Well, how does that help us? Well, it'll help us because if we divide both sides now by two, then on the left we'll end up with X, so let's do that.

Divide by two. So on the left two X divided by two and on the right ten divided by two. Well, we simplify. Well, the reason we divided by two was because we knew that we'd end up with X, and we do because these twos cancel. So on the left we have X and on the right we have ten divided by two, five, and that's the answer. That's the solution of the equation X equals five.

Now, if we want to be sure about that we should check, so let's do that. So we're going to check by substituting the value X equals five into the original equation and check that the left hand side is equal to the right hand side. So what is the left hand side? The left hand side, when we substitute X equals five, is five X, so that's five times five which is twenty-five. So we hope the right hand side is the same as that. The right hand side is three X plus ten. So that's three times five plus ten. Fifteen plus ten is twenty-five. So that's correct. The left hand side is equal to the right hand side so we have got the correct solution.

Unit 5, Example 22: Solving an even more complicated equation

In this example, we're going to solve the equation, seven X minus four equals two X minus fourteen, and the strategy for doing this is to rewrite the equation by doing the same things to both sides to get it into the form, a number, times X equals a number.

So let's look at the equation and see which terms are not in this form. Right, there's a number times X here which is fine, and here's a number on the left hand side which we don't want, and on the right hand side we have this two X here which we don't want and then a number which we do want. So the first step is either to get rid of this minus four or to get rid of the two X, and what we'll do is get rid of the two X.

So we'll start by subtracting two X from both sides. Now on the left hand side we have seven X minus four minus two X, and on the right hand side we have two X minus fourteen minus the two X. And then, after you've done that operation, the next step is to simplify. So we've got on the left hand side seven X minus two X which gives us five X and then a minus four equals, and on the right hand side we've got the two X and the minus two X which cancel, and what we're left with is minus fourteen.

So this is closer to this form that we want to get it into but the term that is, the discrepancy is this term here. So, as the next step to get rid of this term,

we add four to both sides. So on the left hand side we have five X minus four plus four, and on the right hand side we have minus fourteen plus four. As before, when we've done a step, the next thing to do is simplify. So on the left hand side here we have five X, and then the plus four and the minus four cancel, so that's all on the left hand side, and on the right hand side we have minus fourteen plus four which is minus ten.

And now we've got the equation into this form. So the next thing to do is to divide by the coefficient of X. So here we divide by five. On the left hand side we have five X divided by five, and on the right hand side we have minus ten divided by five, and then we simplify. So five X divided by five is just X, minus ten divided by five is minus two, and this is the solution. And it's good practice to check your solution so that's what we'll do now.

So here's the check. So the left hand side of the equation is equal to seven X. Well, X is minus two, so seven times minus two, minus four. That is two sevens are fourteen, so this is minus fourteen. Minus another four is minus eighteen. And on the right hand side of the equation we have two X, and X is minus two, and minus another fourteen. You have two times minus two is minus four, minus fourteen is minus eighteen again, and so it checks all right, and the answer is correct.

Unit 5, Example 24: Solving an equation with fractions and brackets

In this example, we're asked to solve an equation with fractions in brackets. Well, we solve it by using our usual method of a sequence of steps. In each step, we do the same thing to each side of the equation, or we simplify the sides of the equation or we swap the sides. So let's solve this equation. Let's start by writing it down.

So the equation is, well, it's four times X minus five equals X over two plus eight. And the first step is to remove the fractions in brackets. Well, it's usually best to begin by removing the fractions because very often when you remove the fractions you find that you have to put extra brackets in anyway. Well, the fraction that we have to worry about here is this X over two on the right hand side.

So to remove this, to clear the fraction, we're going to have to multiply by two. Now, we always have to do the same thing to both sides of the equation so we're going to have to multiply both sides of the equation by two, and we've also got to be careful to multiply the whole of both sides. So we've got to multiply the whole of this side by two and the whole of this side by two. So let's do that.

So multiply by two. So on the left hand side we're going to have two times four times X minus five, and on the right hand side we're going to have two times, and we're going to have to put this in brackets because we've got to multiply the whole of the right hand side, two times X over two plus eight.

Now let's simplify. Well, on the left hand side we've got two times four, which is eight, into X minus five. And we won't multiply out the brackets yet. We're going to do that in the next step. And on the right hand side -

again, we won't multiply out the brackets - we'll just take out the multiplication sign and write that as two into X over two plus eight. Right, now let's multiply out the brackets.

Well, on the left hand side, we've got eight times X minus five. So we have to multiply each term inside the brackets, that's X and minus five, by the multiplier which is eight. So we're going to have eight times X, which is eight X, and we're going to have eight times minus five which is minus forty. And on the left hand side, well, again, we've got to multiply each term inside the brackets here, X over two and plus eight. Oh, sorry. Okay. And on the right hand side, we're going to have two times X over two which is X, and two times eight which is plus sixteen.

Right, so now what we've got is we've got an equation that has no fractions or brackets. So we can use our usual method to solve it from this point. So what we have to do is to aim for an equation of this form. So a number times X equals a number. And we can get into an equation of this form by adding or subtracting terms to each side of the equation.

Well, let's compare what we've got to the equation that we want to get to. We want a number times X on the left hand side. Well, we've got a number times X here, but we've also got this minus forty, so we're going to have to do something to get rid of that. On the right hand side we want a number. We've got a number here, but we also have the X, so we're going to have to do something to get rid of the X.

Right, let's start by getting rid of the X. Well, we're going to subtract X, just cancel it out, and we're going to have to subtract X from each side. So subtract X. On the left hand side, we're going to have eight X minus forty minus X, and on the right hand side we're going to have X plus sixteen minus X, and we simplify. Well, on the left hand side we've got eight X minus X, so that's going to be seven X, and we're going to have the minus forty, and on the right hand side, well, the X and the minus X cancel out, as we wanted, and we're left with just the sixteen.

So now we're closer to this form that we wanted. The only problem is this minus forty here on the left hand side, so we're going to have to add forty, and we're going to have to add forty to each side. So add forty. Well, on the left hand side we've seven X minus forty plus forty, and on the right hand side we'll have sixteen plus forty, and we simplify. Well, on the left hand side we've got seven X minus forty plus forty, they cancel out, and on the right hand side we've got sixteen plus forty which is fifty-six.

Well, all we need to do now to find the solution of our equation is to divide both sides by the coefficient of X which is seven. So divide by seven. And on the left hand side we'll have seven X over seven, and on the right hand side we'll have fifty-six over seven. And simplify. Well, seven X over seven, that's just going to be X, and on the right hand side fifty-six over seven is eight. So that's the solution of our equation.

Well, let's check to make sure that we've got that right. So we do that by substituting the value X equals eight into the original equation. Let's do that over here.

So, if X equals eight, then on the left hand side where we've got four times X minus five, so that's going to be four times eight minus five, which is four times three, which is twelve, and on the right hand side we've got X over two plus eight. So right hand side is going to be eight over two plus eight, and that's four plus eight which is twelve. So the left hand side is equal to the right hand side when we substitute in X equals eight, so the solution that we found, X equals eight, was correct.

Unit 6, Example 3: Calculating gradients of lines

In this question, we're asked to calculate the gradients of these two lines. So let's look at the graph on the left. This graph has two points plotted on it, minus three, minus four, and the point one two, and using those two points we're going to calculate the gradient. Now, we're going to calculate the gradient using a formula which says that the gradient is equal to the rise divided by the run.

So let me show you what those two things are. I'm going to draw a triangle from this left hand point joining on to the right hand point like that, with a right angle there, and the run is this distance along the bottom, and the rise is this. So the words come from a staircase. You can think of the run as being the distance you go along a step and the rise the distance you go up.

Okay, so the run is equal to the increase in the X coordinate when you move from the left hand point to the right hand point, and that's this distance, and you can see from the X axis that that distance is four. So the run is the increase in X and that's four.

Now, let's talk about the rise. Well, that's the increase in Y. As you go from the left hand point, this one, to the right hand point. So how much does the value of Y increase? Well, you can see again by looking at the axis that it's one, two, three, four, five, six - it's six.

Now, it may be that the numbers are not quite so straightforward as they are in this question, and so there is another way of working out the run. The run is equal to the difference in the X coordinate of the right hand point, the run is equal to the X coordinate of the right hand point minus the X coordinate of the left hand point.

So the run in this case is the X coordinate of the right point one minus the X coordinate of the left point which is minus three, and that again gives you the answer, four. And the rise is the Y coordinate of the right hand point minus the Y coordinate of the left hand point, and that's two minus minus four, and that's equal to six. So, either way, we find that the run is equal to four and the rise is equal to six.

Now the gradient, as I said, is equal to the rise divided by the run. Well that's six divided by four, six over four is equal to three over two, and that's equal to one point five.

Now look at the second graph. Again, we're going to calculate the gradient using the coordinates of these two points. So let's draw the triangle again. So this is the run and this is the rise. And the run is the increase in X. And that's as you move from the left hand point to the right hand point, and you

can see that the increase in X when you move from the left hand point to the right hand point, you go from one to three, so the increase in X is two.

Now what about the rise? Well this is the increase in Y. But here the Y value at the left hand point is four and at the right hand point it's one point five. So actually the Y value decreases rather than increases. That means that the increase is negative, and it goes from four to one point five, so the increase in Y is minus two point five. It's actually a decrease.

Well, you could have worked that out the other way, so let's do that. The run, you can work out the run by taking the X coordinate here and subtracting the X coordinate there. So it's three minus one equals two. And the rise, similarly, you take the Y coordinate at the right hand point and subtract the Y coordinate at the left hand point. So that's one point five minus four and, once again, you get minus two point five.

So now we can calculate the gradient. The gradient equals rise over run, and we plug in those numbers. Minus two point five divided by two and that's minus one point two five.

Now, if you look at those two examples, in the first case the gradient was positive, and that corresponds to the graph rising from left to right, increasing from left to right, and in the example on the right the gradient is negative, and that corresponds to the graph decreasing from left to right.

Unit 6, Example 4: Using the formula for gradient

This question asks you to find the gradient of a line through two points, the point one one and the point three minus one. The question doesn't ask you to draw a sketch of the line but it's often a good idea to draw a sketch so that you can see what's going on. So let's begin by sketching the line.

So let's draw a pair of axes, the X axis and the Y axis, and let's plot these two points. So it's the point one one, this point, and the point three minus one, which is this point. So the line through them looks something like this. So you can see that the line slopes down towards the right, so it's going to have a negative gradient, so you'd expect our answer here to be negative.

Now, you could work out the gradient using rise over run, but there's another way to do it, and that's using this formula. This is what the formula says. It says that the line through the points X one, Y one and X two, Y two has gradients Y two minus Y one over X two minus X one.

So the formula is, essentially, the difference of the Y coordinates over the difference of the X coordinates. Notice it's the Y coordinates on top. And coordinates have to be in the right order. So it's the Y coordinate of the second point minus the Y coordinate of the first point over the X coordinate of the second point, again, minus the X coordinate of the first point. So they have to match up.

So let's use this formula to work out the gradient. So here our first point X one, Y one is one one, and our second point X two Y two is three minus one. So the gradient is, we use the formula, so it's Y two minus Y one over X two minus X one. So Y two, that's the Y coordinate of the second point, which is this, minus one, minus the Y coordinate of the first point, that's one

here, over X two, so X two is the X coordinate of the second point, which is three, minus X one, well, that's the X coordinate of the first point which is one.

So, on the top here, we've got minus one minus one so that's minus two and on the bottom we've got three minus one which is two. So that's minus two over two which is minus one. So the gradient is minus one. And it's reassuring that it's worked out to be negative because we could, indeed, see that the gradient of the line here was a negative.

Now, one thing that you might be wondering is did it matter that we took the first point to be one one and the second one to be three minus one or could we have done it the other way around? Well, let's try it and see what happens.

So this time let's take X one, Y one to be this point, three minus one, and let's take X two, Y two to be this point, one one, and we work out the gradient using this formula. So it's Y two minus Y one over X two minus X one. Well, Y two is a Y coordinate of the second point, which is one, subtract Y one, that's the Y coordinate of the first point, which is minus one, over X two, so that's the X coordinate of the second point, which is one, minus X one, so that's the X coordinate of the first point which is three.

Right, well, on the top here, we've got one subtract minus one, so that's the same as one plus one which is two, and at the bottom we've got one minus three which is minus two. So we've got two over minus two and that gives us minus one. So we've got the same answer. And, in fact, this always happens. For any two points, it doesn't matter which you take to be the first one and which to be the second one, when you work out the gradient using this formula you'll get the same answer either way.

Unit 6, Example 11: Drawing a line from its equation (gradient method)

In this example, we're asked to draw the graph of Y equals three X minus four. Now, if you compare this equation to the equation Y equals MX plus C, you can see that the equation is of that form, so it's a straight line. So the graph is a straight line, okay, and I'm going to draw it using the gradient method. In other words, I'm going to use the gradient of the graph to draw it.

Well, we can read the gradient of the graph off its equation because if you compare the equation to the equation Y equals MX plus C, well, M is the gradient so the gradient is three, so it's a straight line with gradient three, and C is the Y intercept, so the Y intercept is minus four. So Y intercept, minus four. Right, let's go ahead and draw it.

So I'm going to draw some axes using a ruler. And I'm going to mark some scales on the axes, so let's do the X axis first. Doing the tick marks. Right, and now the Y axis. And I'm going to write on the numbers for the scales so zero, one, two, three, four, five. Minus one, minus two, minus three, minus four, minus five. Minus one, minus two, minus three, minus four, minus five. One, two , three. And then remember to put arrows on your axes and label them with X and Y. Right, that's the axes drawn.

So, we said it's a straight line with gradient three and Y intercept minus four. Well, the Y intercept is minus four so that means it crosses the Y axis at minus four. In other words, this point here is a point on the graph, and its gradient is three. So that means that for every unit you go along, you go three units up. So for every one unit you go along, that's one unit along, you go three units up. That's one unit, two units, three units. So this point here is a point on the graph. So let's just draw the line through those two points using a ruler, okay? And that's what the graph looks like, and we label it with its equation, Y equals three X minus four.

Now, once you've drawn a graph, it's a good idea to check that it seems to be correct by working out another point on the graph. So let's do that. Let's see what happens if we put X equals two into the equation. Well, when X equals two, Y will be, we substitute X equals two in here, so Y will be three times two minus four, so that's six minus four, so it's two. So, in other words, the point with X equals two and Y equals two, that's the point two two is on the graph. Well, let's look to see where two two is. That's two units along, two up, so it's here and, indeed, it is on the graph. So it looks like what we drew was correct.

Now, sometimes, you might be asked to draw a graph where this value here, the M value, is a fraction. So, for example, something like this, Y equals minus three quarters X plus one, okay? And the gradient here is a bit more difficult to deal with. Let's look at how we would draw this graph. Well, again, we know it's a straight line, and we know that because the equation is of this form. It's of the form Y equals MX plus C. Just a straight line with gradient minus three quarters and Y intercept one.

Right, let's draw some axes again. Do the tick marks. Put on the numbers. Put the arrows on the axes and label the axes, okay? And we said the Y intercept is one so the graph crosses the Y axis at the value one, that's here, okay, and the gradient is minus three quarters. In other words, for every one unit you go along, well, it's minus so you go down, you go three quarters of a unit down. Well, that's a bit more difficult to draw and it's a bit uncomfortably close. It's going to be uncomfortably close to this point. So let's just try and see if we can make that a bit easier.

Right, we said that the gradient means that for every one unit along, you go three quarter units down. Well, that means that if you go two units along, you must go two times three quarter units down, and if you go three units along, you must go three times three quarter units down, and so on. Now, it would be easiest to think about multiplying by four because that would get rid of the fraction here. In other words, it will cancel out the four and the denominator.

So if you go four units along, you're going to go four times three quarters which is three units down, so let's do that. So four units along, one, two, three, four - let's do that with a dotted line. Four units along and three units down, one, two, three, and I'm using the scales on the axes to see how far to go along and how far to go down. So the graph goes through that point there and then draw a line through the two points and then label it with the equation. So that's Y equals minus three quarters X plus one.

Sometimes you might find that the axes that you draw are not quite big enough for the points that you have to plot so it's a good idea to make sure that you have plenty of space or you might just have to have another go. You might need to extend the axes.

Unit 6, Example 12: Drawing a line from its equation (two point method)

In this question, we're asked to draw the graph of Y equals minus three quarters X plus one. So this equation is of the form Y equals MX plus C, so the graph is a straight line.

Now we could use the gradient method to plot this graph, but I'm going to use a different method, the two point method. The idea of this method is that I find two points that are on the line by substituting values of X into this original equation and calculating the corresponding values of Y.

So I have to calculate two points and let me do the first. Well, because I can choose any two points, it's a good idea to choose simple values of X, and the simplest value of X I could substitute in here is X equals zero because then that term would disappear. So when X equals zero, that term disappears and I just have Y equals one. So, in that case, the point is zero one.

So that's a point on the Y axis because the first coordinate is zero at height one and actually I could have read this straight off from the equation of the line because one here is the Y intercept and that tells you the height and the Y axis that the line crosses. So that's my first point.

What about a second point? Well, as I said, I should try and choose values of X which make my calculation as simple as possible. If, for example, I chose the value X equals one, then this would be minus three quarters and I would have fractions still. But if I were to choose X equals four, so when X equals four, well, what happens now is that we get Y equals minus three quarters times four plus one.

Because I chose X equals four, this part of the calculation turns out to be straightforward. This is just minus three. It gives me an integer plus my one and that's equal to minus two. So the point in this case is four for the X coordinate and minus two for the Y coordinate. So now I have two points that are on the line, zero one and four minus two, so I'm going to plot those on the axes and draw a straight line through them.

So let me draw some axes, there's my X axis and my Y axis, and I'll mark some scales. And on the Y axis, so let me put arrows on and numbers, five, one, two, three, four, five. Zero at the origin, minus one, minus two, minus three, minus four, minus five. Minus one, minus two, minus three, minus four, minus five. Okay, so I have my axes ready. Let me label this the X axis and the Y axis.

So where are these two points? The first point, zero one, is here on the Y axis. So one is the Y intercept. And the second point, well, that's four across, and it's minus two, so it's two down, so this point is there, four

minus two. And now we join those two points by a straight line. And there's my straight line.

Okay, let me mark it with the equation of the graph, Y equals minus three quarters X plus one, and then I'm finished except that I should do a check. I'm going to calculate a third point on the graph and make sure it is on the graph. So here's a check.

Well, if I'm going to calculate another point on the graph, then to make life simpler I should choose a value of X which makes this calculation as simple as possible and, well, X equals four was good last time so let's try X equals minus four. So when X equals minus four, Y equals, well, now it's minus three quarters. I'm going to put that in brackets because I'm going to be multiplying by minus four plus one. Well, minus three quarters times minus four is three plus one and that gives me four.

So the point, this third point, is, well, its X coordinate is minus four and its Y coordinate is four. So let's see if that lies on the graph. The X coordinate is minus four, the Y coordinate is four, so, yes, I think that lies on the graph. So that confirms I have the correct line.

Unit 6, Example 14: Finding the equation of a line from the gradient and a point

In this question, we're asked to find the equation of the line that has gradient minus two and passes through the point one three. Well, when you're asked to find the equation of a line, it's a good idea to start off by drawing a rough sketch, so let's do that. So let's draw a Y axis and an X axis, some tick marks.

Let's label the axes, okay, and we're told the line passes through the point one three, so let's plot that point. So along one, up three, it's here. And we're told that it's got gradient minus two, so that means that for every one unit you go along you go two units down. It's down because it's minus two. So one unit along, two units down, that takes us to this point. So our line passes through these two points here. Let's draw it in, something like that.

Okay, let's go ahead and find the equation of the line then. So this is what we know. We know that the equation of the line is, well the equation of any straight line, except for vertical ones, is Y equals MX plus C, it's of that form. Where M is the gradient and C is the Y intercept.

So if we know the values of M and C, then we just need to substitute them in and that gives us the equation of the line. In this case, we do know the value of M because that's the gradient minus two, but we don't know the value of C, but let's go ahead and substitute in the value of M.

So the gradient is minus two. So the equation is Y equals, substituting minus two in for M, minus two X plus C. So now to find the equation of the line, we've got to find the value of C, and we've got to do that using this information, the information that the line passes through the point one three.

Well, because the point one three lies on the line, it means that the coordinates one three satisfy this equation. Let's write that down. So the

point one three lies on the line. So its coordinates satisfy the equation. So that means if we substitute X equals one and Y equals three into this equation it will be satisfied. So let's do that.

So Y is three, so we've got a three on the left hand side, equals minus two times X, and X is one, so it's one, plus C. And let's simplify that, that will give us three equals minus two times one is minus two plus C, and now we've got an equation in C that we can solve to find the value of C, and to do that all we have to do is add two to each side. So adding two to the left hand side will give five, adding two to the right hand side will give C. So the value of C is five. And now we can go ahead and substitute it into our equation to find the final version of the equation.

So the equation of the line is Y equals, from here minus two X, plus C, and C we've got now is five, so the equation is Y equals minus two X plus five. Well, that's answered the question, but it's always a good idea to do a check, and in this case we can see that what we have is the equation of a line with Y intercept five. Let's have a look at our diagram to see if that looks as if it corresponds to what we've got there. Well, it does indeed look like the Y intercept of this line is five. So that's reassuring. It seems that our equation is likely to be correct. So let's write the check down. The Y intercept of the sketch appears to be five.

Unit 6, Example 15: Finding the equation of a line through two points

In this question, we're asked to find the equation of the line that passes through the points one two and three five. So let's draw a rough sketch of the graph.

So there's my X axis and my Y axis, and I'm going to do some tick marks. I'll need five of these for those points. So the first point is one two, that's about there, and the second point is three five, so that's about there, and I'll draw a straight line roughly through those two points. Okay, so that's roughly what the graph looks like. Now I know that the equation of a straight line is of this form, Y equals MX plus C, so my answer should be of that form. M is the gradient of the line and C is the Y intercept of the line, that's where the line crosses the Y axis. So first I'm going to find the gradient using the information that's given in the question.

So the gradient is, now there's a formula for the gradient, and the formula tells you that you should take the difference between the Y values, that's the second Y value minus the first Y value, and divide it by the difference between the X values, that's the second X value, minus the first X value. So the gradient is the difference between the Y values, five minus two, the second minus the first, and the second X value minus the first X value, that's three minus one. So five minus two over three minus one, that's three over two.

So I know what the gradient is, the value of M, and so I can write down the equation of the line with that information in it. So the equation is Y equals three over two X plus C. So we've found half the information, we found M and now need to find the value of C, the Y intercept. Well, what do we

know? We know that these two points, one two and three five, lie on the line, so the X and Y coordinates of those points both satisfy this equation. So let's pick one of the points, and I'm going to pick one two, and substitute those values into this equation, and that'll tell me what the value of C is.

So I'm going to use the fact that the point one two lies on the line. So if I substitute the values in Y equals two, this is three over two, times the X value which is one plus C. Well, okay, that's two is equal to three over two plus C. And now if I subtract three over two from both sides, two minus three over two gives me a half, the three over two's cancel, and I get the value of C to be equal to a half. So the equation of the line is Y equals three over two X, plus its value of C we've just found, which is a half. So there's the equation of the line.

Well, if possible, you should always check your answers, so what checks can we do here? Well, the M value, the slope three over two, the gradient, is positive, and that tells me that the line should slope uphill from left to right, and it does, so that's a check. This value, a half, is the Y intercept, so the graph should pass through the Y axis at height a half, and it roughly does, so that's a good check. But the other thing I can do is to substitute the coordinates of the second point into the equation and see if they satisfy the equation.

So let's try that. When X is equal to three, well I should get the value Y equals five, let's see if I do. Y is equal to three over two times three, the X value, plus a half. Well, that's three over two times three is nine over two, plus a half. That's ten over two, and that's five, which is exactly the value that we wanted, so that check worked, and it looks as if our equation is correct.

Unit 6, Example 16: Finding the equations of horizontal and vertical lines

In this question, we're asked to write down the equations of two straight lines. The first line is parallel to the X axis and the second line is parallel to the Y axis, and you'll see that these are both easier special cases.

So in this first case I'm going to draw a quick sketch of the graph. There's my X axis and there's my Y axis. And I'm told that the line is parallel to the X axis and through the point zero two, so let's mark the point zero two on the Y axis and draw a line parallel to the X axis. So there's the line parallel to the X axis through the point zero two.

Now, if you think about all the points that are on this line, what do they have in common, well, they're all a height two above the X axis. So all points satisfy Y equals two. And that's the equation of this line. It's as simple as that, Y equals two.

Now, if you were to try and write down the equation of this line using the standard form of a straight line, Y equals MX plus C, then you would find that the gradient of this line is zero, M equals zero, so you would get Y equals C where C takes the value two, so you'd get exactly the same equation. So in this case, let's write a concluding sentence. The equation is

Y equals two. And, in general, all lines parallel to the X axis have an equation of this form, Y equals a number.

Let's look at Part B. Here we're asked to write down the equation of the line parallel to the Y axis and through the point minus three one. So let's draw a sketch again. There's my Y axis, here's my X axis, and I need to plot the point minus three one, so minus one, minus two, minus three and one on the Y axis and the point minus three one is here. So let's mark that, minus three one. I need to draw the line parallel to the Y axis through that point so I need to draw a vertical line. So there's my vertical line.

Now, let's argue in the same way as we did in Part A. All points that are on this line have one thing in common, their X coordinate is minus three. So for all those points the X value is minus three, and that's the equation of this straight line. So the equation is X equals minus three and, in general, all lines parallel to the Y axis will have an equation of this form, X equals some number.

Now, you might think that you could work out the equation of this line by using this general form, Y equals MX plus C, but, in fact, if you try to do that with a vertical line, then when you try to calculate the gradient, you'll be dividing by zero and so it won't work. Vertical lines don't have a gradient so their equation is of this different form, X equals some number.

Unit 7, Example 2: Making a variable the subject of another equation

In this question, we're asked to make R the subject of this equation. In other words, we're asked to rearrange the equation so that it's in this form. It looks something like this. It's going to have R by itself on the left hand side and R should not appear at all on the right hand side. And the reason that we might want to rearrange the equation into this form is that, when we've got the equation into this form, if we know the values of all the other variables, Q and P in this case, then we can just substitute them into the expression on the right hand side and then use that to work out the value of R.

Now the way to rearrange an equation, to make a particular variable the subject, is to use the same method that you use to solve an equation, treating the variable that you want to be the subject, which is R in this case, in the same way that you would treat the unknown when you solve an equation. So let's go ahead and do that. The first thing to do is to write down the equation. So the equation is P equals Q over R plus one.

Now when you're solving an equation, or when you are rearranging a formula, as in this case, the first thing to do is to remove any fractions or brackets that you have. And in this case we've got a fraction on the right hand side, and it's got a denominator, R plus one. Now R plus one just represents a number, R represents the number so R plus one represents a number, so we can clear this fraction by multiplying both sides by R plus one. So let's do that. Multiply by R plus one. Right, so on the left hand side we've got P, and we're going to multiply by R plus one.

Now remember we're multiplying by the whole of R plus one, so we're going to need to enclose it in brackets to make that clear. It's P times all of R plus one, not just P times R plus one, okay, and we remember that when we do something to an equation, when we do something to both sides of an equation, we must do it to the whole of both sides. So we're going to multiply the whole of the right hand side by R plus one. So that we are making it clear that it is the whole of the right hand side, we're going to enclose the right hand side in brackets. And because we're multiplying by the whole of R plus one, we're going to enclose R plus one in brackets. Right, so we've now multiplied by R plus one, and the reason we did that was to try to remove this fraction here.

So the next stage is to simplify. Well, let's just keep the P times R plus one on the left, for the moment, because I'm going to multiply the brackets in a minute. And on the right hand side, well, the reason that we multiplied by the R plus one was because we knew that they would cancel and remove the fraction, and that's just what happens. To go through that in more detail, if you did this multiplication you would end up with Q times R plus one on the top and the R plus one would cancel with the R plus one on the bottom, so all together we're left with Q.

Right, the next stage in solving an equation or rearranging a formula is to multiply out the brackets, so let's do that. So we've got P times the first term, which is R, so that will give us PR, and P times the second term, which is plus one, so that will give plus P, and on the right hand side we've still got the Q. Right, so now we've got an equation that's got no fractions or brackets, so we're ready to go into the next stage of rearranging this formula. Now remember that what we're aiming to do is to get the equation into this form.

Now if you were solving an equation then what you would do is you would get all the terms containing the unknown on the left and all the other terms on the right, and I'm going to do something very similar here. This equation here just has one term that contains R, the variable that I want to be the subject. And what I'm going to try to do is to keep this term here on the left and to get everything else over on the right. In other words, I'm going to try to get something like this, PR equals something, okay, because PR is the term I've got here, and the reason I'm going to do that is that then all I will have to do is divide by P, divide both sides by P, to get R by itself.

So I want to keep this PR on the left, and I want to make sure that everything else, all the terms that don't contain R at all, are over on the right, and to do that you can see that what I need to do is to subtract P from both sides because that will cancel out this plus P on the left. So let's subtract P. Okay, so on the left we'll have PR, and then we'll have plus P minus P, so that's all we'll have, and on the right we'll have Q minus P, and now I've got the equation into the form that I wanted it, and as I said to get the equation into the final form that I want with R by itself on one side and not at all on the other side all I've got to do is divide by P.

Now I have to be just a little bit careful here because you have to remember that you can't divide by zero. So the step that I'm going to do here is valid for all values of P except zero. The other steps that I did up here were valid for any value. So I'm going to emphasise that by writing down here divide

by P assuming that P is not equal to zero. And that will give R on the left, because PR divided by P is just R, and on the right it will give Q minus P over P.

So I've done what I said I wanted to do, I've rearranged the formula into this form, it's got R on the left by itself, everything else on the right, and R doesn't appear at all on the right, and this formula can be used for any value of Q and P except when P is equal to zero.

Unit 7, Example 3: Rearranging the equation of a line

In this question, we're asked to rearrange the equation minus five X plus three Y equals four into the form Y equals MX plus C. So we're being asked to write it in the form of the standard equation of a straight line. So what we have to do is to get Y, this term, all by itself on the left hand side of the equation. In other words, we're trying to make Y the subject of the equation.

Well, in order to do that we have to get this term, minus five X, away from the left hand side and onto the right hand side, and we can do that by adding five X, which we do to both sides of the equation. So if we add 5X to the left hand side, we're just left with three Y, and if we add five X to the right hand side, we get five X plus four.

So now we've made progress because the only term in the left hand side is three Y, but what we want is to get just Y on the left hand side, and we can do that by dividing by three. So divide by three. Now if we divide the left hand side by three, we get Y, and if we divide the right hand side by three, well, we get the whole of the right hand side, five X plus four, all divided by three.

So we've got to the stage where now Y is the subject of the equation. Y is all by itself on the left hand side, but the right hand side is not yet in the form MX plus C. Now to get the right hand side in that form we have to expand this fraction. So let's write that here. Expand the fraction and what do we get? We get Y equals, well, this is five X over three plus four over three, and that's nearly in the correct form. We just have to rewrite five X over three as five over three times X plus four over three. And now we've succeeded in rearranging that equation into the form Y equals MX plus C, five over three X plus four over three.

The next part of the question asks us to, hence, write down the gradient and Y intercept and sketch the line. So we compare the form of the equation we have here with Y equals MX plus C, and that tells us that the gradient is, M is the gradient, M is five over three, so the gradient is five over three. And the Y intercept is, well, the Y intercept is C, this constant, and that's four over three.

So I've answered the first part. I've written down the gradient of the Y intercept. Finally, I'm asked to sketch the line. Well, if you know the gradient and you know the Y intercept, then you can use the gradient method to sketch the line. But I'm going to use the two point method. To use the two point method, I've got to find two points on the line and then I

can join those two points by a straight line, and I'm going to choose convenient values of X and substitute them into this equation. Well, the simplest value of X to choose is always X equals zero, so when X equals zero, well, in that case Y is equal to four over three. So in that case the point is zero, four over three, and that's a point on the Y axis.

What other values shall I take for X? Well, I could take X equals one. When X equals one, what do I get? Y equals, well, five over three times one is five over three. Let's write it down, five over three times one, plus four over three, and that's equal to five over three plus four over three, which is equal to nine over three, and that's three. So this point is, now, the X coordinate was one and the Y coordinate was three.

Now, you can use those two points and plot them on a graph and join them up by a straight line, and here it is. So this is a sketch of the line through the two points, zero four over three and one three, and we can mark this with the equation of the line, and we could write Y equals five over three times X plus four over three, or we could use the equation in the question, minus five X plus three Y equals four. They both describe this straight line. So minus five X plus three Y equals four and we're done.

Unit 7, Example 7: Finding a highest common factor

This example is to find the highest common factor of two terms. So I'll start off by writing down the two terms. The terms are, well, the first term is six A B to the seventh C squared, and the second term is nine A squared B to the fifth. Now we write down what the highest common factor is. Right, when I'm doing this, I like to proceed from left to right in the left hand term and build up the highest common factor as I go along.

So starting off with the coefficient on the left, we have a six which is two three's here and a nine which is three three's so the highest common factor is three. Here on the left we have an A term here and an A squared in the right hand term and the highest common factor of those is A because A is the biggest power of A that divides both of them.

So on the B term, we have a B to the seventh here and a B to the fifth here. So what's the highest common factor of those? Well, we know that B to the seventh equals B to the fifth times B squared. So B to the fifth definitely is a factor of this term, and it's also a factor of this term because it's B to the fifth times B squared. So B to the fifth is, in fact, the highest common factor of those two.

So what else is in the left hand term? Well, there's a C squared here, and there's no Cs in the right hand term, so it doesn't appear in the highest common factor. So this is the highest common factor of these two terms.

So now the next part of the example is to write each term in the form the highest common factor times something. So the terms can be written as, so the highest common factor which is three A B to the fifth and times something.

So what do we need to get this first term? Well, for the coefficients, we've got three and we want a six so we need to have a two there. We've got an A. We want an A, so we don't need to have any more As in this term. We've got a B to the fifth, and we want a B to the seventh, and we know that B to the fifth times B squared gives B to the seventh so we must have a B squared here. So we haven't got any Cs here. We want a C squared, so we need it in this term here. So this is of the form the highest common factor times something else, which is another term.

So what's happened to the second term? So we write down the highest common factor A B to the fifth times, so we've got a three, we want a nine, so we need another three. We've got an A. We want an A squared, so we need another A. We've got a B to the fifth, and we want a B to the fifth, so we don't need to add anything else, and that is in that form of the highest common factor times something and so we're done.

Unit 7, Example 8: Taking out a common factor

We're asked to factorise this expression. When you're asked to factorise an expression, you should take out the highest common factor. So let's begin by finding the highest common factor.

So the highest common factor is, we need to find the highest common factor of this expression. In other words, we need to find the highest common factor of the two terms here; this term and this one.

Let's begin by looking at the numbers. Well, there's a three in the first term, but there's no number at all in the second term, so there's going to be no number in the highest common factor.

Next, let's look at the R's. Well, there's an R in the first term, and there's an R in the second term, so there will be an R in the highest common factor.

Finally, let's look at the S's. Well, there's an S cubed in the first term, and there's an S in the second term, so the highest power of S that divides both the S cubed and the S is S. So the highest common factor is RS.

Now let's take the highest common factor out of the expression. So let's begin by writing down the expression. That's three RS cubed plus RS. And the first step here is to write each of our two terms in the form the highest common factor RS times something. So let's begin by doing that for the first term. So it's RS times something, and this must be something such that when we multiply it by RS you get the whole term three RS cubed back again.

Now we've got the number three in here, and we haven't got a number so far here, so we're going to have to put three in. Next, let's look at the R's. We need an R, and we've got one, so that's okay. Next, let's look at the S's. We've got an S cubed here. We've only got one S so far, so we're going to have to multiply by S squared. So that's the first term. The first term is RS times three S squared.

Next, let's look at the next term, the second term, so we've got plus in here, and we're going to want to write it in the form RS times something, and the something must be such that the RS times the something gives us the whole

term here. Well, that's straightforward here because we just need a one. RS times one gives us the RS. Now we can go ahead and take out the RS as a common factor.

So we write our common factor, RS, at the front, and we open a set of brackets, and inside the brackets we write what's left of each term. So we're taking the RS away from the first term. In other words, we're dividing the first term by RS, and that will leave us with three S squared, and we've got plus, and what's left of the second term when we divide by the RS, it's just one. So it's one. So the factorisation of our expression is RS times three S squared plus one. If you want, you can check the answer by multiplying out the brackets again.

RS times three S squared plus one equals, well, you multiply the multiplier by each of the two terms inside the brackets individually. So RS times three S squared will be three R, S times S squared is S cubed, so that's the first term, and then you multiply the RS by the next term which is plus one, and that's just going to give plus RS. And you can see that this expression here is the original expression that we started with, so the factorisation was correct.

Unit 7, Example 9: Factorising an expression containing minus signs

In this question, we're asked to factorise this expression which has three terms. So the first thing we're going to do is to find the highest common factor of these three terms.

So the highest common factor is, well, let's look at the numbers first. They're three, six and three, and three is certainly a factor of all of them, and in fact it's the highest factor of all of them, so the numbers give us three. The M's? Well, we have M cubed, M squared and M to the fourth, and M squared divides into all of them, and in fact it's the highest power of M that divides into all three of these. So we get M squared from the M's. So our highest common factor is three M squared.

Well, now we want to take out three M squared from this expression. So let's write the expression down. Plus three M to the fourth. And we want to take out three M squared as a factor of each of these terms. So we're going to write each of these terms in the form three M squared times something.

So let's look at the first. The first term is three M cubed. So we have a three, but then we have a three here already, and we've got an M cubed, but we've got an M squared here, so to make M cubed we need times M. So that's the first term dealt with.

Well, now we've got six M squared. So we want three M squared as a factor. We've got six over here. We want six, and we've got three, so we want times two. Then we've got M squared over here, but we've already got an M squared here so we don't need any more powers of M, so that's that second term done.

Now let's do the third term, plus three M squared times something. Well, we've got three, over here, and we've got three on the right. We've got an M to the fourth over here, but we've only got M squared here, so we need to multiply this by M squared.

So now we've written each of our three terms as three M squared times something. So we can now take out the factor, three M squared, and we do that by writing three M squared and opening brackets, and then three M squared times M, well, that will give us an M inside the brackets. Minus three M squared times two. Well, that will give us a minus two inside the brackets. Three M squared times M squared, that'll give us plus M squared inside the brackets. There, we've factorised our expression. It's three M squared, the highest common factor, times this expression with three terms in brackets.

Well, now, you can check your work by taking the final expression and multiplying it out. So let's do that. So here's our check. Three M squared is the multiplier multiplying the brackets M minus two plus M squared. And we have to take the multiplier and multiply each of the terms inside the brackets. So the first term is three M squared times M, which is three M cubed. Next, it's three M squared times minus two. So that's minus, well, minus two times three is minus six M squared, and then three M squared times M squared which is three. M squared timed M squared is M to the fourth. And that's the expression that we started with, three M cubed minus six M squared plus three M to the fourth, so our factorisation was correct and we're done.

Unit 7, Example 10: Factorising efficiently

This example is to factorise the expression minus eight X squared plus two X plus two XY, and in doing this factorisation we're going to use the quicker method where, although we go through the same steps, we don't write down as much.

So the first step is, as before, to find the highest common factor of these three terms. So what is the highest common factor? Look at the numbers first. We have a two, a two and an eight, and the highest common factor is two. The X terms, we have an X, an X and an X squared which gives a highest common factor of an X. And, in the Y terms, we have a Y here but we don't have Y's in this term or this term so there is no Y in the highest common factor. So this is the highest common factor. And that's going to be the multiplier outside the brackets in the factorised expression. So let's write a bracket in.

So what do I need here in order to give this expression, this term on the left? So we've got a two, we want a minus eight, so we need this to be minus four. We've got an X, and we want an X squared, so we want this to be an X.

So go on to the next term. So we've got a two, we want a two, so there's no number term. We've got an X and we want an X. So there's no other terms involved so there must be a one here in order to say that this one times this is what we want.

So let's go on to the third term. We've got a two, we want a two, so there's no number term. We've got an X and we want an X, so there's no X's. We've got no Y's here, but we want a Y, so we need a Y here. So this is our factorised expression.

So let's check this now by multiplying out the brackets. So here's the check. So we'll write out two X into minus four X plus one plus Y, and the multiplier is two X and the first term is minus four X, so the term is two times minus four is minus eight. X times X gives X squared. Two X times the second term which is one just gives us two X. And two X times the third term, X plus Y, gives plus two XY, and this expression is exactly the same as what we were given, so the check is satisfied and we're done.

Unit 7, Example 13: Making a variable the subject of an equation

In this example, we're asked to make C the subject of this equation. And we're going to do that by using our usual method of a sequence of steps, in each of which we do the same thing to each side of the equation or simplify the sides of the equation or swap the sides.

Now the first thing to do always is to check for fractions and brackets, and to remove them if there are any. In this case there aren't any, so we're okay we can go on to the next stage. The next thing to do is to aim for an equation like this. We want something like this. We want an expression times the required subject, which in this case is C, equals some expression. So we want to aim for this. And we're going to do this in two steps.

First of all, we're going to make sure that all the terms that contain the required subject C are over on the left hand side, and that all the other terms, all the terms that don't contain the required subject C, are over on the right hand side. So let's do that. Let's begin by writing down the equation. So it's two C minus A equals BC plus one.

Now we want all the terms containing the required subject on the left. Well this is a term containing the required subject, and it's on the left, so that's okay. But this is a term that doesn't contain the required subject, so we're going to have to remove that term. And of course we're going to do it by doing the same thing to each side of the equation, so we're going to have to add A. On the right hand side, remember what we want on the right hand side is only terms that don't contain the required subject. Well, this term does contain the required subject, so we're going to have to remove that term by subtracting BC from both sides. This term is okay because it doesn't contain the required subject.

Right, well let's begin by removing this term; we're going to subtract BC from each side. Well, on the left we're going to have two C minus A minus BC, and on the right we're going to have BC minus BC, and they'll cancel out, plus one, so we're left with the one. Right, now we're a bit closer to this, but what we want now is to remove this term from the left hand side, because this is a term that doesn't contain the required subject C, so we don't want such a term on the left hand side. So we're going to remove this term, and we're going to do that by adding A to both sides. So

add A. Well on the left we're going to have the two C, minus A plus A, so they cancel out, and then minus BC. And on the right hand side we'll have the one, and we're adding A, so that will be plus A.

We've now succeeded in making sure that the left hand side only contains terms containing the required subject C. This term contains C and this term contains C. And we've succeeded in making sure that the right hand side only contains terms that don't contain the required subject C. But we haven't quite got to this form yet. The right hand side is okay because it's certainly an expression, but the left hand side isn't quite in the form the expression times C. But we can rewrite the left hand side in that form by taking out the required subject C as a common factor. Let's do that.

Well, on the left hand side we're going to get C. We've got two C here, so what's left is the two, and then we've got minus BC, we've taken out the C, so we're left with minus B, and on the right hand side we've got one plus A. So we've now succeeded in getting an equation of this form. We've got an expression, here, times C, here, equals an expression over here. My C and my expression are in a different order from here, but that doesn't matter.

Right, so all we need to do now to make the C the subject is to divide both sides of the equation by the expression that multiplies the required subject C, and that expression is two minus B. Well if I divide by two minus B, on the left hand side I'll have C times two minus B divided by two minus B, so that will just give C as we wanted, and on the right hand side I'll have one plus A all divided by two minus B.

So I've now succeeded in making C the subject of the equation, because you can see that I've got C by itself on one side of the equation and on the other side I've got an expression that doesn't contain C at all. So C is certainly the subject of the equation. And there's just one more thing to notice about this, and that's that in the last step I divided by two minus B. Now you can't divide by zero, so that means that the equation that I ended up with isn't valid when two minus B is zero. In other words, it's not valid when B is equal to two.

Unit 7, Example 14: Solving simultaneous equations by substitution

In this question we're given these two simultaneous equations for the unknowns A and B; nine A minus two B equals twelve and B equals five A minus fourteen. And I'm going to call these Equation One and Equation Two. And the aim is to rearrange these equations so that we end up with an equation which only has one unknown in it, and then we can solve that equation.

Now, the second equation is quite nice because B is expressed in terms of A, and so we can substitute that equation directly into the first equation, and the result of that is that we'll get an equation just involving A. So let's substitute for B in one.

Well, one starts nine A minus two B, but now we're substituting for B, five A minus fourteen, and that's equal to twelve. But, okay, we've now got an equation with just A in it but we've got to expand this bracket and simplify.

So let's expand the bracket. That's nine A, minus two times five A is minus ten A. Minus two times minus fourteen, that's plus twenty-eight, is equal to twelve, and then simplify.

So nine A minus ten A, well, that's minus A, plus twenty-eight is equal to twelve. And now I can solve for A if I can remove the twenty-eight from this side, and I can do that by subtracting twenty-eight from both sides. So on the left I'll be left with minus A and on the right I'll have twelve minus twenty-eight is minus sixteen.

Well, to get an equation with A on the left, I can multiply both sides by minus one. So I multiply by minus one, and that changes both sides. A is equal to sixteen. So I've managed to find the value of one of the unknowns, A is equal to sixteen.

Now, to find B, I can use Equation Two and substitute this value for A in Equation Two. So I'm going to substitute A, substitute for A in Equation Two. And I get B is equal to five times A. Well, that's five times sixteen minus 14, and I think I know that five times sixteen is eighty minus fourteen, and that's equal to sixty-six.

So I've found A and I've found B so the solution is A equals sixteen and B equals sixty-six.

Well, I can check my solutions by substituting them back into the original equation. So I'll do that. Check. Well, let me check equation one first. On the left hand side, we have nine A minus two B is equal to nine times sixteen minus two times sixty-six, and nine times sixteen is a hundred and forty-four, and I can do two times sixty-six, that's a hundred and thirty-two. If I do that subtraction, that's twelve. And that's exactly what I should have on the right hand side of Equation One.

And Equation Two, well, on the left hand side I have B which is equal to sixty-six, and on the right hand side I have five A minus fourteen, and that's equal to, well, we did this sum earlier on, five times sixteen minus fourteen, that's eighty minus fourteen is sixty-six, as required. So our check's done and we've solved the equations.

Unit 7, Example 15: Solving more simultaneous equations by substitution

In this example, we're asked to solve a pair of simultaneous equations using the substitution method. Let's begin by writing down the equations.

So the equations are: the first one is two X plus four Y equals eight and the second one is minus three X plus five Y equals minus one. And I'm going to label the first Equation One and the second one Two. Now the first step in the substitution method is to choose one of the equations and to choose one of the unknowns in that equation and to make that unknown the subject of the equation.

So let's do that. I'm going to choose the first equation, and I'm going to choose to make X the subject of the equation. So I'm going to make X the subject of Equation One. Right, well, this is Equation One. It's two X plus four Y equals eight. And I want to make X the subject. So I want to

remove this term in Y from the left hand side, and I'm going to do that by subtracting four Y from each side. So subtract four Y. And on the left hand side, that's just going to leave me with two X, and on the right hand side I'm going to have eight minus four Y.

Now, to make X the subject of the equation, all I have to do is divide both sides by two. On the left hand side, I'm going to have X and on the right hand side I'm going to have eight minus four Y over two. And then I'll simplify. So I'll have X, well, I expand the fraction here, and that will give me eight divided by two which is four and minus four Y divided by two which is minus two Y.

Right, so I've now got X as a subject of this equation. And the reason for doing that is that I can now take this equation and I can use it to substitute for X into the other equation. I started with Equation One. This is a rearrangement of Equation One, and I'm going to use it to substitute into the other equation which is Equation Two.

So I'm going to substitute into Equation Two, and I'll do it over here. Well, Equation Two is this one. So it's minus three X, but I'm substituting for X using this equation, so I'm going to get minus three and replace X by four minus two Y, and then I've got plus five Y equals minus one.

Now I have an equation that only involves Y and I'm going to solve this equation to find the value of Y. Well, the first thing to do is to multiply out these brackets here. So I multiply out. Well, I've got minus three times four, and that's minus twelve, and I've got minus three times minus two Y, and that will be plus six Y, and then I've got plus five Y equals minus one.

The next step is to simplify the equation by collecting these like terms here. So that will give me minus twelve plus six Y plus five Y is plus eleven Y equals minus one.

The next thing I need to do is to remove this minus twelve from the left hand side by adding twelve to each side of the equation. So that will leave eleven Y on the left and on the right we'll have minus one plus twelve so that will give eleven. So to find the value of Y all we have to do now is divide by eleven, and that will give Y equals one.

So we've found the value of Y and all we need to do now is to find the value of X, and we can do that by substituting into, well, we can do it by substituting into either of the original equations we had or, in fact, it's even easier to use this equation because this equation actually expresses X in terms of Y. Let me call this equation, Equation Three, and I'm going to substitute into Equation Three. And that will give X equals four minus two times Y, and we know that Y is one now. So it's four minus two times one. So it's four minus two which is two. So I've now found the value of X and the value of Y so I can write down what the solution is. The solution is X equals two, Y equals 1.

Now, it's always a good idea to check the solution by substituting back into the original equations, so let's do that. Let's look at what happens if I substitute these values into the left hand side of the two equations. So substitute into Equations One and Two. Well, substituting into the left of Equation One will give me this, the left hand side of Equation One is two X

plus four Y, and that will be, well, X is two, so it's going to be two times two plus four Y, Y is 1, so it's plus four times one. So that will be two times two, which is four, plus four times one which is four, so it's four plus four which is eight. And now we look to see whether that's the same as the right hand side of Equation One, and it is, so that seems to be correct.

Let's just try Equation Two as well. So Equation Two, the left hand side was minus three X plus five Y. I substitute in my values, X equals two and Y equals one, so that will give me minus three times two plus five times one. So that's minus six plus five which is minus one. And I checked to see whether that is the same as the right hand side of Equation Two and it is.

So substituting back in has confirmed that these values here, X equals two and Y equals one, do form the solution of the system of two simultaneous equations.

Unit 7, Example 16: Solving simultaneous equations by addition

This example is to solve these simultaneous equations: the equations three A plus two B equals twenty-four and three A minus two B equals thirty-six. Now, you could solve these equations using the substitution method as we've done before, but because of these equations being in a nice form, having a plus two B and a minus two B, it is quicker to use the elimination method, and that's what we're going to use here.

So I'll number the equations, One, and this one equation, Two, and we'll start off by adding the equations. So add one and two. So on the left hand side we have three A plus three A which gives us six A and two B plus minus two B which is going to cancel and on the right hand side we have twenty-four plus thirty-six which gives sixty.

Now, we proceed to solve this equation so we just divide by six to give A on the left hand side and sixty divided by six is ten. This gives the value of A. To find the value of B that corresponds to this, we could substitute back into either of the equations. In this case, we're substituting to Equation One. And we get three A, so that's three times ten, plus two B equals twenty-four. Simplifying that, it's three tens are thirty plus two B equals twenty-four.

Now, we need to get rid of this term on the left hand side for thirty so we'll subtract thirty from both sides. And on the left hand side we'll have thirty minus thirty which will cancel, plus two B, and on the right hand side we'll have twenty-four minus thirty which is minus six. So, to solve this equation, we just divide by the coefficient of B which is two. So that will give B equals minus three, and that's the solution: A equals ten and B equals minus three.

So now we need to check our solution. So as a check we'll substitute the values into the equations. So here's the check. So three A plus two B is three times ten plus two times minus three, well, that's thirty minus six is twenty-four, and the right hand side of this Equation One is twenty-four, so that checks out.

So let's look at the other equation. Three A minus two B is at three times ten minus two times minus three. Well, that is thirty as before. Minus two times minus three is going to be plus six, so that's going to be thirty-six, and that is the right hand side of the Equation Two, so that also checks out and we're done.

Unit 7, Example 18: Solving simultaneous equations by elimination

In this question, we're asked to solve the simultaneous equations: three X plus two Y equals eight and minus five X plus three Y equals minus seven, and I'll label these One and Two. And we're asked to solve these equations by the elimination method, and that means that we're going to eliminate one of these two unknowns, and I can choose to eliminate either Y or X, and I'll choose to eliminate Y. And the way I do that is to multiply each of these equations by a number, and I do it in such a way that the coefficient of Y is the same after I've done the multiplication.

So if I were to multiply the first Equation by three, then the coefficient of Y would be six, and if, at the same time, I multiplied this equation by two, then the coefficient of Y will also be six.

So let's do that. So three times Equation One will give me, well, three times three is nine, so that's nine X, plus three times two is six, six Y equals twenty-four. And now I'm going to do two times Equation Two, and I'll get minus ten X plus six Y equals minus fourteen. So, now, as I wanted, the coefficients of Y are the same in both equations.

So I can subtract one equation from the other and eliminate Y. So I get nine X, on the left, minus minus ten X is equal to twenty-four minus minus fourteen. Now let me simplify that. Well, nine X subtract minus ten X is nine X plus ten X so that's nineteen X, and twenty-four subtract minus fourteen is twenty-four plus fourteen, and that's thirty-eight.

So now I can solve for X by dividing both sides by nineteen and the result is X is equal to thirty-eight divided by nineteen which is two. So now I've found the value of the variable X. Well, to find Y, I can substitute for X in one of these two equations, and I'm going to choose to substitute in Equation One, so substitute for X in Equation One. Now, in Equation One I get, well, three X is three times two plus two Y is equal to eight.

Now, simplifying that, that's six plus two Y equals eight, and now to get two Y by itself I can subtract six from both sides. So two Y is equal to eight minus six is two, and that gives me Y equals one if I divide both sides by two. So the solution is X equals two and Y equals one.

Well, as usual, we can check the solution by substituting X equals two and Y equals one into our original equations so let's do that. Check.

Let's do Equation One first. Well, three X plus two Y, substituting these values we've found, is equal to three times two plus two times one, three times two is six, two times one is two, and that sum is eight, and that's what we should have on the right hand side of that equation.

And now let’s check Equation Two. On the left hand side is minus five X plus three Y, and when we substitute these values, this is minus five times two plus three times one, and that’s equal to, well, that’s minus ten plus three, and that’s equal to minus seven, and that’s what we wanted. So we’ve solved those equations.

Unit 8, Example 5: Finding angles related to a garden shed

In this problem we have to find four unknown angles: Alpha, Beta, Gamma and Delta. And we’re given various pieces of information on the diagram. That angle is sixty-five degrees. We’re told that the line AB is parallel to the line DC, and we’re also told that the line AE is parallel to BC and they’re both vertical. Also this angle, the angle AED, is a right angle, so the line segment ED is horizontal. Well, I’ll work my way around the diagram finding these angles one by one starting with the angle Gamma, which is adjacent to the angle sixty-five degrees that we’re given.

Now the Gamma and sixty-five degrees are angles on a straight line.

That means they add up to a hundred and eighty degrees.

So subtracting sixty-five degrees from both sides, I have Gamma equals a hundred and eighty minus sixty-five, which is one hundred and fifteen degrees.

On next to the angle Beta, well we said that AB was parallel to DC, and BC intersects them both, so the angles Beta and sixty-five degrees here are corresponding angles.

That means that they must be equal, so Beta is equal to sixty-five degrees.

Next, we look for the angle Alpha. In order to find that, I will first extend line segment BA up to the left and label some point to the left of A, F.

Well we know that AE and BC are parallel lines, and BA intersects them both. So that means that the angle FAE and Beta, up here, are corresponding angles.

And so they’re equal. So the angle FAE is equal to Beta, which we previously found to be sixty-five degrees.

Now to find Alpha, we notice that Alpha and the angle we’ve just found, FAE, are angles on a straight line.

And that means that Alpha plus angle FAE equals one-eighty degrees. FAE, we just found to be sixty-five degrees, so Alpha plus sixty-five degrees equals one hundred and eighty degrees. So Alpha is the one hundred and eighty degrees minus sixty-five degrees, which is one hundred and fifteen degrees.

Finally, I need to find the angle Delta, and to do that I’ll make another extension by extending the line segment ED across and allowing it to intersect with the line BC at a point which I’ll call G. Now ED is horizontal and BC is vertical, so this is a right angle. On this triangle, I know the

angle is sixty-five degrees, and I know this angle is ninety degrees, and I know that the sum of the angles in a triangle adds up to one hundred and eighty degrees.

So that means the angle CDG, plus sixty-five degrees, plus ninety degrees, equals a hundred and eighty degrees. So the angle CDG is equal to the one hundred and eighty degrees, minus sixty-five degrees, minus ninety degrees, which comes out at twenty-five degrees.

Finally, to find Delta, I notice that the angle CDG and Delta are angles on a straight line.

So Delta plus the angle we've just found, twenty-five degrees, is equal to one-eighty degrees, and that means that Delta is one hundred and fifty-five degrees.

And there it is: we've found the values of Alpha, Beta, Gamma and Delta.

Unit 8, Example 7: Showing that triangles are congruent

In this question, we're asked to show that two triangles are congruent. Recall that two triangles are congruent if one can be placed exactly on top of the other with one of the triangles been flipped over if need be. There are several tests to determine whether two triangles are congruent, and these are detailed in the unit. And to determine which of these tests to use we need to look at the information we have about each of the triangles. So let's do that in this particular case.

In the triangle on the left, we know the angle at A, and we know the angle at B, and we know the length of the side between them. Thus we know an angle-side-angle arrangement. However, in the other triangle, we know these two angles but not the length of the side between them; instead we know the length of this side. So here we have an angle-angle-side arrangement. But I do notice that the length of this side is equal to the length of that side. So in order to be able to match up the information between the two triangles it would help to find the size of the angle Y here.

Well, to do that, we recall that the interior angles of any triangle, so in triangle XYZ in this case add to a hundred and eighty degrees degrees. So the angle at Y here, that is the angle XYZ, will be equal to a hundred and eighty degrees subtract the other two angles. A hundred and eighty degrees subtract eighty-eight degrees and subtract thirty degrees, which equals sixty-two degrees. So I'll just mark that on the diagram for myself so that I can now see that the angle-side-angle arrangement that I have in the left hand triangle is matched exactly by the angle-side-angle arrangement I have in the triangle on the right hand side. So what I need to do now is to confirm that in detail.

So the first part of the test is to state that the angle at A, that is angle CAB, is equal to sixty-two degrees, and the angle at Y, that is the angle ZYX, is equal to sixty-two degrees, so that confirms that angle CAB is equal to angle ZYX. Now moving on to the sides, the length of the side AB is equal to four, and the length of the side YX is equal to four, hence the length of the

side AB equals the length of the side YX. And finally the second angle, the angle at B, that is the angle ABC, equals thirty degrees, and the angle at X, that is the angle YXZ, equals thirty degrees, so confirming that angle ABC equals angle YXZ. Hence the angle-side-angle test is met, so we can write, hence, by angle-side-angle, or ASA, triangle ABC is congruent to triangle YXZ.

Note that the letters here are written in the order to show how the corresponding angles, ABC here and YXZ, match up, and hence how the corresponding sides match up. In this case, it would have been possible to prove that the two triangles are congruent by using the angle-angle-side test. To do this, we would have found the angle at C in the left hand triangle there instead of calculating the angle Y in the right hand triangle.

Unit 8, Example 12: Using Pythagoras' theorem to find a shorter side

In this problem, we're asked to find the length of the third side of a right angled triangle. One side has length five metres, next to the right angle, and the hypotenuse, opposite the right angle, has length thirteen metres. Well, for a general right angled triangle, with side lengths A and B next to the right angle and C for the hypotenuse, we know that A squared plus B squared equals C squared. That is known as Pythagoras theorem.

Well, in the current case, we have A equals five and C equals thirteen. Both of the lengths are in metres but I won't include the metres in this statement of Pythagoras theorem.

Well, we're looking for the length of the third side, let that length be B metres, and then the statement of Pythagoras theorem for this case: A squared plus B squared equals C squared, A is equal to five, plus B squared, is equal to C squared where the C is equal to thirteen. Subtracting five squared from both sides, that is B squared, equals thirteen squared minus five squared, or working out the squares, that's one hundred and sixty-nine minus twenty-five, which is one hundred and forty-four.

Well, that's the square of B, so B itself will be the square root of one hundred and forty-four. And I take the positive square root here because I'm looking for the length of a side of a triangle. The square root of one hundred and forty-four works out to be twelve. So what have we found? The length of the third side is twelve metres. Looking back at the diagram that seems to be a reasonable outcome because the side length that we're looking for has got to be slightly less than the hypotenuse of length thirteen metres, so twelve metres looks like a reasonable answer.

Unit 9, Example 2: Multiplying out two brackets

In this question, we're asked to multiply out various pairs of brackets, and in each bracket, we've got two terms. In this one, we've got an X and a plus one, and in this bracket, we've got an X and a plus two. When you're multiplying out brackets like this, you have to multiply each term in the first bracket by each term in the second bracket. So here you have to multiply

the X by each term in the second bracket, that's by the X and by the plus two, and then you have to multiply the one by each term in the second bracket, that's by X and by the plus two. So there're four multiplications to do all together.

Now, you can do the multiplications in any order that you like, but it's a good idea to be systematic and to do them in the order that I've just shown you. Which is, first of all, start with the first term in the first bracket, and then multiply it by the first term in the second bracket. Then, again, the first term in the first bracket, multiply it by the second term in the second bracket, and then go on to the second term in the first bracket. First multiply it by the first term in the second bracket and then by the second term in the second bracket. And one way to remember this order is to use the acronym FOIL.

This stands for First-Outer-Inner-Last. So it means, first of all, you multiply the first two terms together, then O for Outer means you multiply the two outer terms together, then I for Inner means you multiply the two inner terms together and, finally, L for Last means you multiply the two last terms together. So let's do that with the first example.

Well, it's X plus one, times X plus two. First of all, we multiply the two first terms together, so that's X times X, which is X squared. Then we multiply the two outer terms together, so that's X and plus two, and that will give us plus two X. Then we multiply the two inner terms together, that's plus one and X, and that gives plus X. And, finally, we multiply the two outer terms together, that's plus one and plus two, and that gives plus two. And the last step is to simplify, so we collect like terms. Well there's only one term in X squared, so that just stays as X squared, and then we've got two terms in X. We've got plus two X and plus X, so all together we'll have plus three X, and finally we've got the constant term plus two. So that's the answer to this one. That's what we get when we multiply out these two brackets.

Well, we can use exactly the same methods to multiply out these two brackets here, so let's do that. So it's X minus one, times X minus two. And we'll use FOIL again. So first of all we multiply the two first terms together, that's X and X, giving X squared. Then we multiply the two outer terms together, that's X and minus two, giving minus two X. Then we multiply the two inner terms, that's minus one and X to give minus X. And finally the two last terms, that's minus one and minus two, giving plus two. We collect like terms, and again we've got a term in X squared, just X squared, and we've got minus two X minus X giving minus three X, and finally we've got the constant term, plus two, and that's that one, that's the answer to this one.

In Part C, we've got X plus one squared. Now, if you've got something squared, it just means you multiply the something by the something. So let's begin by writing that down. X plus one squared is just the same as X plus one times X plus one, and then we multiply out in exactly the same way that we used above. So, first of all, we multiply the two first terms together. So that's X times X, giving X squared. Then we multiply the two outer terms together, that's X and plus one, giving plus X. Then we multiply the two inner terms together. That's plus one times X, giving plus

X again. And, finally, we multiply the two last terms together, that's plus one and plus one, giving plus one. And we simplify. So we're going to have X squared, plus X, plus X, gives plus two X, plus one at the end. So that's the answer to that one.

Now let's look at this one here. Again, we've got two brackets to multiply out. That's A plus two B, times three C, minus D. Well we multiply the first term here, A, by the first term here, three C, and that's going to give three AC. That's the first terms. Now we look at the outer terms. That's A times minus D. That's going to be minus AD. Now we look at the inner terms, that's plus two B times three C. Well they're going to multiply together to give plus six BC. And finally we look at the last terms, that's plus two B, times minus D, and that's going to give minus two BD.

Right, now we look forward to seeing whether there's any like terms that we can collect. Well, we've got an AC here; no more AC's. We've got an AD here; no more AD's. We've got a BC here, and that's not BC, so there's nothing that we can do here, there's no like terms to be collected, so that's the answer to multiplying this one out.

Finally, let's look at Part E. So this is N minus one and N plus one. Use the same method, multiply the first terms together, so that's N and N, giving N squared. Look at the outer terms, that's N and plus one, giving plus N. Look at the inner terms, that's minus one, times N, giving minus N, and finally the last term, minus one and plus one, multiplying together to give minus one. We simplify by collecting like terms where we're going to have N squared. Then we're going to have plus N minus N, so they cancel out. And finally we're going to have the constant term, minus one. So that's the answer to the last part.

Unit 9, Example 3: Solving simple quadratic equations

In this question, we're asked to solve these two quadratic equations. And the first one is X squared minus nine equals nought. And there's no X term in this quadratic equation so it should be more straightforward to solve, and in fact we can solve it by adding nine to both sides, so let's add nine. Well, the left hand side we then get X squared and on the right hand side equals nine. So if X squared is equal to nine, X must be equal to three or minus three. Those are the two square roots of nine. If you multiply three by itself you get nine and if you multiply minus three by itself you get nine because minus times a minus makes a plus. So we've solved that equation.

The other equation is X squared minus ten equals nought. It's a very similar equation so let's apply the same strategy. We'll add ten to both sides. So we're going to get X squared is equal to ten. Now ten isn't a perfect square, as nine was, so that one was easy to solve, but if X squared is equal to ten, then the values of X must be, well the square root of ten, the positive square root of ten, or X equals minus the square root of ten. Well, those are exact answers, but if you need the answer as an approximate decimal, then you could use your calculator and find that the square root of ten is equal to three point one six two, say, and that's to three decimal places.

Unit 9, Example 4: Factorising a quadratic expression

In this question, we're asked to factorise the quadratic expression: X squared plus six X plus eight. And that means we're asked to write it as a product of two simpler expressions, and each of these simpler expressions should have a term in X in it and a constant term in it. Well, since because we've got X squared on the left here, X squared is X times X, so our terms in X will be an X in this bracket and an X in that bracket, and then we have to work out what terms to put after the X.

Well, if we multiply these constant terms together then we're going to get the answer eight. So if we want these constant terms to be integers, and we do, then we should look at the possible factor pairs of eight. So the factor pairs of eight. Well, one times eight makes eight, so that's a factor pair, and two times four makes eight, so that's a factor pair, and at the moment I'm just going to look at the positive factors.

Well if you put constant terms in here and here, and then multiply out the brackets, multiplying this by X you'll get a term in X, and this by X you'll get another term in X, and you want these two terms in X to add up to six X. So what we want to do is to choose, if possible, a factored pair of eight whose sum is six. So the pair with sum six. Well, the first pair, one and eight, that doesn't have sum six, but the second pair, two four, does have sum six, so the pair with sum six is two four. Well now that we've found a pair whose product is eight and whose sum is six, we can write down the factorisation, and it's X squared, plus six X, plus eight, is equal to X, plus two, times X plus four, and that's our factorisation.

Well, whenever you've factorised an expression, it's important to check that your factorisation works, and you can do that by multiplying out the products. So let's do that check. You should at least do it mentally but let's actually do the calculation here. X plus two times X plus four. Well, multiplying X by X we get X squared. Multiplying X by four we get plus four X. Two times X is two X. And two times four is eight. Well let's collect those like terms together, X squared plus six X plus eight, and that's correct so our check works. You can see, as we multiply those together, how these constants emerged. The two times four, this was a factor pair of eight, gave us the eight, and then the four and the two added together, the four X plus two X add together to give six X, so that's how that works.

Now you can use that strategy to try to factorise any quadratic expression. But if these signs are negative, rather than positive, then you may have to consider factor pairs with negative numbers in them as well as positive numbers.

Unit 9, Example 5: Factorising a quadratic expression

In this question, we're asked to factorise the quadratic expression X squared minus five X plus six, and as you've seen that means that we want to write it as the product of two smaller expressions, each of which contains a term

in X and a constant term. Now, if you think about multiplying out these two brackets here, then we can see that we're going to end up with an X squared term, and that means that there must be an X in each of these brackets. So the factorisation will look something like this. And we've got to find constant terms in here such that when we multiply the two brackets together we get this quadratic expression, and we're hoping that we can find integers to fill in here.

Now let's think about the numbers that we could fill in here. When we multiply out these brackets, then as part of that multiplication we're going to have to multiply the two last terms together, and when we do that we know that we're going to end up with a six. So the two terms that we fill in here must multiply together to give six. Now let's think about the middle terms when we multiply out. Well when we multiply out the outer terms, this one times this one, we're going to get a term in X, and when we multiply out the inner terms, this one times this one, we'll get a term in X, and we know that those two terms must add together to give minus five X. So you can see that the number that we put in here and the number that we put in here must add together to give minus five.

So, for the numbers that we put in here, we're looking for them not only to multiply together to give six but also to add together to give minus five. So let's think about what numbers they could be. Well, if they're going to multiply together to give six, a positive number, then either both the numbers here must be positive, or they must both be negative. Now if they are both positive then they're certainly not going to add together to give the negative number minus five. So we can see that what we're looking for is two negative numbers in here. Two negative numbers that multiply together to give six and add together to give minus five.

So the way to find these is to look at the factor pairs of six. And we're only interested in the negative ones as we said. So the negative numbers that form factor pairs of six are minus one and minus six, and minus two and minus three, and we want to find a pair that add together to give minus five. Well, these two add together to give minus seven so that's no good, but these two do indeed add to give minus five, so it looks like this is the pair we're looking for. So the pair with sum minus five is minus two and minus three. So it looks like our factorisation is X squared minus five X plus six is X, and in this gap we fill in the minus two, so it's X minus two, and in this gap we fill in the minus three, so it's X minus three.

Now when you factorise a quadratic expression you should always check by multiplying out, even if you only do it mentally. It's easy to get your signs wrong and it's very quick just to do a check to make sure that you've got it right. So let's do a check.

So X minus two, X minus three, multiply the first terms, X times X gives X squared. Multiply the outer terms, that's X times minus three, which gives minus three X. Multiply the inner terms, minus two times X is minus two X. Multiply the last terms, that's minus two times minus three, which is plus six. Now simplify where we've got our term in X squared. Collect these like terms in the middle, as usual, so that's minus three X minus two X, which is minus five X. And we've got our constant term, plus six. And now we look back to see whether that is the quadratic that we were trying to

factorise, and it is, so we were correct. That's the correct factorisation. And you can now see why we needed these two numbers here to multiply together to give the minus five.

The two numbers minus two and minus three ended up giving us this minus three X and the minus two X, and then they ended up adding together to give us the minus five X, which is what we wanted.

Unit 9, Example 6: Factorising a quadratic expression

In this problem, we have to factorise the quadratic expression X squared minus seven X minus eight. That means we need to write it as the product of two simpler expressions, each of which contains an X and also a constant term. Now in this quadratic expression the constant term minus eight is negative, and that means if we look for products of the terms in here, they cannot both be positive or both be negative; they will have to have opposite signs. So in looking for factor pairs of minus eight, we'll write out all of the possibilities.

Well, one possibility is one and minus eight. Another is two and minus four. Then there's four and minus two, and finally eight and minus one. There's all the integer possibilities for factor pairs of minus eight. And now what I'm interested in is the factor pair that adds together to give minus seven.

And by inspection of the four factor pairs, only the first, the one and the minus eight, will add to give minus seven.

So the one and the minus eight are the numbers that we want to feed in here to fill the gaps in the two simpler expressions. In other words, X squared minus seven X minus eight factorises as X plus one times X minus eight. That's the answer. As a check, we can multiply X plus one by X minus eight and see that we get back to the X squared minus seven X minus eight. So here we go, X plus one, times X minus eight, equals X times X, gives X squared. X times minus eight gives minus eight X. Plus one times X gives plus X. Plus one times minus eight gives minus eight. Which simplifies to X squared minus seven X minus eight. And that is, indeed, the quadratic expression that we started from, so the check works.

Unit 9, Example 7: Solving a quadratic equation

In this problem, we're asked to solve the equation X squared, minus seven X, minus eight, equals zero, and the method suggested is factorisation. So let's factorise the quadratic on the left hand side. This expression X squared minus seven X minus eight was factorised in a previous tutorial clip. The factors were X plus one and X minus eight. So having factorised that we have X plus one times X minus eight equals zero.

Well, we have a situation here where there's a product of two numbers equal to zero. That means that either one of the numbers must be zero or the other of the numbers must be zero. So either X plus one equals zero or X minus eight equals zero, and each of those are equations that can be solved

in one step. So if X plus one equals zero then X is equal to minus one. On the other hand, if X minus eight is equal to zero then X equals eight. And that's the solution of the equation that was given.

We can check that by feeding the values that we found back into the original quadratic equation. When X is equal to minus one, X squared minus seven X minus eight is equal to minus one all squared, minus seven time minus one, minus eight, which is equal to one plus seven minus eight which is equal to zero. So that satisfies the equation X equals one is a solution.

On the other hand, when X is equal to eight, the left hand side of the equation, X squared minus seven X minus eight, is eight squared, minus seven times eight, minus eight, that's equal to eight squared, sixty-four, minus seven times eight, fifty-six, minus eight, and again that's equal to zero, showing that X equals eight is also a solution of the original equation, so both of the checks work.

Unit 9, Example 8: Factorising a general quadratic expression – first method

In this example, we're asked to factorise a quadratic expression. Now this example is a bit different from the ones that you've seen so far because the coefficient of X squared isn't one; it's two. And not only that, but two isn't a common factor of the expression. If two were a common factor of the expression, then you could take it out, and you'd be left with a quadratic expression in which X squared did have coefficient of one, and then you could use the usual methods to factorise it. But, as I've said, in this case two isn't a common factor of the expression, and so it is a more tricky expression to factorise. But let's look to see what we can do.

So the basic idea is, just as before, we want to take our quadratic expression, and we want to write it as a product of two smaller expressions, each of which has a term in X and a constant term. Now before what we did was we put an X in here and an X in here, and we started from there, but you can see that if we do that then when we multiply out the first terms of these factors, then we're going to get an X squared, and that's not what we want, we want a two X squared. So the thing to do is to put a two X in here and an X in here and then when we multiply out we will get the two X squared. And then we need to put constant terms in here, and we're hoping that we can put in integers, so that when we multiply out the two factors we get this quadratic expression.

Now let's think about what these integers that we're going to put here have to be. Well, just as before, you can see that when multiply out the last terms of these two factors we're going to have to get minus six. So the two numbers that go here will have to have the property that when you multiply them together you're going to get minus six. So, just as before, we're going to have to look at the factor pairs of minus six. So let's write those down. Factor pairs of minus six. Well what could we have? We could have minus one and six, or we could have one and minus six, or we could have minus two and three, or finally two and minus three.

Now before all you had to do was to choose the factor pair whose sum is the coefficient of X, which is minus one in this case, but now it's not quite so simple. Let's think about what happens when you put two numbers into these gaps. When you multiply out, well let's think about the outer terms, you're going to get two X times this number, and then you're going to add that to what you get when you multiply out the inner terms which will be this number times X. So in fact the coefficient that you're going to get over here is not going to be just the sum of these two numbers, but in fact it's going to be two times this number plus this number, and it all gets a bit complicated. Really, the simplest thing to do is just to try the various possibilities and see what you get and keep going until you get the right answer. You'll see how it works out as I do it.

So let's begin by looking at this factor pair here. Let's just rip that bit off. Let's look at what happens if I put these numbers into the gaps. So that would be minus one and plus six. Let's look at what happens when I multiply out and see whether I get the right answer. Now I don't have to consider multiplying out the first terms because I know that I'm going to get two X squared, as I want, because that's why I chose to put the two X and the X in here. And, similarly, I don't have to consider what happens when I multiply at the last terms, the minus one and the six, because I chose the minus one and the six so that they would multiply together to give the minus six over here.

So in checking whether this factorisation here is correct, all I've got to do is to consider the outer terms, two X times plus six, and the inner terms, minus one times X. So let's do that. Well the outer terms, that's two X times plus six, which is twelve X all together, and for the inner terms, it's minus one times X, which is minus X, so I've got twelve X, minus X, which is eleven X. So when I multiply this out the term in X will be plus eleven X, and that's not what I want, I want minus X, so this factor pair isn't correct. Let's put a cross there to indicate that.

So let's now rub these two numbers out and try the next ones, one minus six, and this factor pair is just the same as the first one except that the signs are the other way round. This one's positive and this one's negative, and that's worth noticing for a reason that you'll see in a minute. So let's put the numbers into the gaps. So plus one here, minus six here, and let's see whether this factorisation is correct.

Just as before, we don't need to consider the first terms and the last terms because we know that they are going to be correct; I just need to look at the outer and inner terms. So the outer terms will be two X times minus six which will give me minus twelve X, and the inner terms will be plus one times X which will give me plus X. So I'm going to have minus twelve X plus X which will give me minus eleven X, and that's not what I want because I'm looking for minus X, so this factor pair doesn't give me the correct factorisation.

Now there's something to notice about what I've just done, and that is that this factor pair gave me minus eleven X, and this factor pair gave me plus eleven X, so the term in X that I got from this factor pair was the negative of the term in X that I got from this factor pair, and that's because this factor pair is just the same as this factor pair but the signs are different, so you've

got minus one here and one here, six here and minus six here, and that will always happen. It will happen for these ones over here, this factor pair here is the negative of this one, so the term in X that I get for this one will be the negative of the term in X that I get for this one, and you'll see that happening in a minute.

So the next thing that we need to do is to try this factor pair. So let’s rub out the numbers that we've got here and put this factor pair in instead. So that’s minus two in here and plus three in here, and when I multiply the outer terms, that’s two X times plus three, I'm going to get plus six X, and when I multiply the inner terms, that’s minus two times X, I'm going to get minus two X, so that’s plus six X, minus two X, which all together will give me plus four X, and that’s not what I want because I want minus X. So this factor pair doesn’t give me the correct factorisation either.

Well what about this factor pair? Well, from what I said before, we know that because this factor pair gave me plus four X that this factor pair is going to give me minus four X, which again is not what I want, so I can immediately tell that this factor pair isn't correct.

Now it looks like I've exhausted all the possibilities, but in fact I haven't, because I could put each of these four factor pairs into the gaps in the other order. So, for example, instead of putting this minus two here and the three here, I could put the three here and the minus two here, and that would give me different results because this two X isn't the same as the X. So there's four more possibilities that I can write down here; the same factor pairs but in the opposite order.

And we now need to go through each of these ones. So let’s begin by rubbing out these numbers so that we can try the next factor pair. So that’s plus six here and minus one here, and when I multiply the outer terms, that’s two X times minus one, I'm going to get minus two X, and the inner terms will give plus six times X which is plus six X, so that’s minus two X plus six X which gives me plus four X, and again that’s not what I'm looking for, unfortunately, because I'm looking for minus X. So this factor pair doesn’t give me the correct factorisation.

Well, next, what about this factor pair, minus six one? Well, this factor pair gave me plus four X, so this factor pair is going to give me minus four X, and we’re looking for minus X. So this one is no good either; it’s not going to give the right factorisation.

So the next thing to do is to try this one. So let’s rub out the numbers here and do that. So that’s a plus three here and a minus two here. Now this time I'm going to get two X times minus two, for the outer terms, and that’s going to give me minus four X, and I'm going to get plus three times X, for the inner terms, and that’s going to give me plus three X. So I'll have minus four X plus three X which will give me minus X and at last that is what we’re looking for. So it looks like this is the correct factorisation. Let’s write that down. So the factorisation is two X squared minus X minus six equals two X plus three times X minus two.

Now that should be correct because that’s what we just worked out up here, but it’s always a good idea to a check, so let’s do that. So two X plus three, X minus two, the first terms, that’s two X times X which will give me two

X squared; outer terms, that's two X times minus two, which is minus four X; inner terms, well that will be plus three X. Last terms, that's going to give me minus six. Collect like terms, that will give me two X squared, minus X, minus six, which is indeed what we were looking for. So the check seems to verify that the factorisation is correct.

There's one more thing to notice about this example and that is that there's something about it that makes it easier than it might have been. When we looked at the two X squared here, and we had to decide what term in X we were going to put here and what term in X we were going to put here, there was only really one possibility, at least if wanted to use integer coefficients. The only possibility was a two X and an X. Now that's not always the case for other quadratics.

For example, look at this one: four X squared plus seven X minus fifteen. Suppose you wanted to factorise this one, so again you'd have to write it as a product of two smaller factors, each of which has a term in X and a constant term. Now, in this case, there's two possibilities for what the terms in X could be. You could have a four X here and an X here. Or, alternatively, you could have a two X here and a two X here. So what you'd need to do for this quadratic is you'd need to go, for each of these two possibilities you would need to go through the same process that we did up here. So it would take a bit more time to do it. However, you will find that with practice this method will become quicker for you to do.

There is also another method that you can use to factorise quadratic expressions which is particularly useful for more complicated ones like this, and you'll see that in the next tutorial clip.

Unit 9, Example 9: Factorising a general quadratic expression – second method

In this question we are asked to factorise the quadratic expression two X squared minus X minus six. Now you've already seen in a previous tutorial clip one way of factorising this expression by looking at the various cases, but here I'm going to show you a different method which always finds you a factorisation if there is one but it's a bit mysterious how it works. The main thing is to follow the steps as I go through them.

So let me remind you first of the form of a general quadratic expression, AX squared plus BX plus C. A, B and C are the coefficients there, and in this case you can see that A is equal to two, B the coefficient of X is equal to minus one and C the constant coefficient is equal to minus six. So those are our values of A, B and C in this case.

Now the method starts by writing down factor pairs. So let's write that, factor pairs. Now usually you would write down factor pairs of minus six, in the previous examples you've seen, but here we're going to write down the factor pairs of, well, this coefficient times this coefficient, that's A times C, which in this case is two times minus six, which is minus twelve. So let's write down the factor pairs of minus twelve. And they are minus one times twelve, one times minus twelve, minus two times six, two times minus

six, minus three times four and three times minus four. So there's quite a few of them.

Okay, so we've written down the factor pairs of minus twelve. Now we have to find the pair, in this method we have to find the pair, which add to give, well it’s B, add to give B equals minus one. So that’s similar to previous method. Now we scan through the pairs until we find a pair, and it’s the last pair, three minus four. So the pair we want is three minus four. So we've written down the factor pairs of minus twelve and we've picked out the pair which adds to give minus one.

What we do with this pair is to rewrite the original quadratic expression. So we rewrite or rearrange the quadratic expression in the following way. So we start with our quadratic expression, two X squared minus X minus six, and this is how we rewrite it. The only change we make is to the middle term, the X term, so we write down two X squared. Now the middle term has coefficient minus one, and we know that minus one is the sum of these two numbers, three and minus four, so we introduce terms in X with coefficients three and minus four. So we write down plus three X, minus four X. So together those produce that term minus X. And then we add minus six on the end. We don’t make any change to that.

So we started with a quadratic expression with three terms and we've split up the middle term; we have four terms now. The next thing we do is to group these four terms in pairs. So I'm going to put a brace under those two terms and a brace under those two terms, then I'm going to look at each of these pairs of terms. What I'm going to do is to look at them and try and extract common factors.

So let’s look at this first pair. Well there's no number that’s a common factor because you’ve got a two here and a three there, but you can take out a common factor of X because X divides into two X squared and X divides into three X. So let’s do that. Two X squared minus X minus six is equal to, I'm taking out a common factor of X, so X times two X gives me two X squared, and X times three gives me three X.

Now let’s look at the next two terms. Well there's no common factor of X here but four and six have a common factor of two, so let’s take that out. Well actually there are two minus signs as well, so I'm going to take the minus sign out and have minus two, and then I'll need minus two into two X to give me the minus four X and plus three to give me the minus six. Well now let’s look at what we've got. We've got X times two X plus three, minus two, times two X plus three again, so we have the same factor here.

So actually this is a common factor and we can take it out in the usual kind of way. It looks a bit bigger than the normal common factor we have but we can take it out. So we can write this as X minus two, with brackets round it, all into two X plus three. And you can check that that would work by imagining multiplying this out, you'd have X times all of that, X times two X plus three, minus two times all of that, minus two times two X plus three.

So now we've achieved the factorisation that we wanted. The factorisation is two X squared minus X minus six is equal to this product: X minus two times two X plus three.

Well you should always check your factorisations so let's do that. We write down the product, X minus two, times two X plus three, and do the usual multiplication. This first term, X times two X, so it's two X squared, X times plus three, well that's plus three X, minus two times two X, that's minus four X, and minus two times three, that's minus six. So now let's collect the like terms. Well this is equal to two X squared, and then three X minus four X is minus X, and then we have minus six. And that's exactly the quadratic expression we started with. So our factorisation is correct.

Well there's just one thing you might have been wondering and that is at this stage where we took minus X and replaced it by three X minus four X, just before we grouped the terms, what if we'd put these the other way round, minus four X and then plus three X. Well, you can try that yourself, and you'll find that the method still works.

Unit 9, Example 11: Simplifying algebraic fractions

In this question, we're asked to simplify the following algebraic fractions. Here's the first one, A squared over A to the power five, A to the fifth. And the way we're going to do that is to look for common factors of the numerator and the denominator so we can cancel them. That is we divide these common factors into both the numerator and the denominator.

So let's look at this first one. Well, in the numerator we have A squared, and A squared divides into A to the power fifth. Because A to the fifth is equal to A squared times A cubed. It's a bit of working over there. So let's write this down and do that cancellation, A squared over A to the fifth. Well if we divide A squared into the top we get one, and if we divide A squared into the bottom we get A cubed. So there's our answer, one divided by A cubed.

Now let's look at the second example, this looks more complicated: sixty P cubed Q divided by thirty-five, P to the fifth R. Well let's start by looking at the numbers. I'll write it down again. Sixty P cubed Q over thirty-five P to the fifth R. Well both sixty and thirty-five have a common factor of five so we can cancel that. Five into sixty goes twelve times and five into thirty-five goes seven. Now let's look at the letters. Well here we have P cubed and here we have P to the fifth, and we know that P to the fifth is equal to P cubed times P squared.

So we can cancel that P cubed to give one on the top and P squared on the bottom. Well now we have this cube but there's no other term in Q to cancel with it in the bottom, and similarly with R. So that's the end of the cancellation and we can write down the result as, in the numerator twelve times Q, and in the denominator we've got seven times P squared times R. Seven P squared R, so that's the answer to that one.

Now this third one looks a bit more complicated: two X squared plus six X divided by X squared minus nine. And when you first look at it you might think there are no common factors. But each of these is a quadratic expression, and it can be factorised, so we'll do that and then we'll look for common factors. Well two X squared plus six X is a quadratic expression with no constant term at the end, and that means that you can factorise it by

taking out a factor of X. In fact, you can take out a factor of two X in this case. So this is two X into X plus three. If you multiply that out you get two X squared plus six X.

And the denominator, X squared minus nine, is also a special kind of quadratic. It's a difference of two squares because it's of the form X squared minus three squared. So that factorises as X minus three times X plus three. Well now that we've factorised the numerator and denominator you can see that there is a common factor, X plus three is a factor of both, so we can cancel that and write down the answer as two X divided by X minus three. And that's the answer.

Unit 9, Example 12: Adding and subtracting algebraic fractions

In this problem, there are three expressions which have to be turned into single algebraic fractions. The first is three over X minus two over X. Now these have the same denominator, X. So we can simplify simply by writing a single fraction with denominator X and then subtracting the numerators, three minus two. That gives a final answer of one over X.

In Part B, the expression given is three over A plus one, plus two over A plus one, minus one over A plus one. Again, the denominator is the same in each case; here it's A plus one. So I can write a single fraction with denominator A plus one and add and subtract the numerators; that's three plus two minus one. And then working out three plus two minus one, that's four, over A plus one is the final single algebraic fraction.

Lastly, in Part C we have A over X plus B over Y, and here the denominators are different; there's X in the first case and Y in the second. So we need a common denominator for both of the fractions, and one possibility that will always work is to simply multiply the two individual denominators together, X times Y. So I want to express each of my two individual fractions with denominator XY.

So the A over X, I can multiply both the top and the bottom of that by Y to get AY over XY. And in the second case, where I again want a denominator XY, I can achieve that by multiplying both the top and the bottom here by X; that gives me BX over XY. Now I have a situation where each of the two individual fractions has denominator XY so I can add the numerators to obtain AY plus BX on the top and XY on the bottom.

This is certainly a single algebraic fraction. You should always check that it's not possible to simplify such an answer because it's required in its simplest form, if possible. But here there are no like terms and no possible factorisation so it cannot be simplified any further.

Unit 9, Example 13: Multiplying algebraic fractions

In this question, we're given two algebraic expressions, and we're asked to write each of them as a single algebraic fraction. And each expression is made up of two algebraic fractions multiplied together. Well, to multiply algebraic fractions you just do the same as you do it when you're

multiplying numerical fractions; you multiply the numerators together and you multiply the denominators together. So let's look at the first example and do that.

Well it's A over two X, times B over three Y. We multiply the numerators together, so that will give us A times B, and we multiply the denominators together and that will give six XY. When you've obtained your final answer you should always look to see whether there's any simplification that you can do, and in this case there isn't, there are no factors that can be cancelled on the numerator and the denominator, for example. So let's go on to the next example.

So that's three X over five Y, times Y, times Y plus one, over X squared. And again we multiply the numerators together, and that will give three XY, Y plus one, and we multiply the denominators together, and that will give five X squared Y. And I'm writing it as X squared Y rather than YX squared because it's usual to write letters in alphabetic order in products unless there's a good reason not to do that. Well, again, once you've got this answer, we look to see whether there's anything that can be simplified, and in this case there is.

For example, you can see that X is a factor of the numerator and it's also a factor of the denominator so that X can be cancelled. So we cancel this X and we get one, we cancel the X squared and we're left with X, because X squared divided by X is just X. And there's another factor that can be cancelled on the numerator and denominator, and that's this Y. There's a Y on both the numerator and denominator. So cancel that Y, we get one; cancel that Y, and we get one. We divide top and bottom by Y.

Right, and what are we left with? Well the numbers that we've got on the numerator are three times one times one, which is three, and we've also still got this Y plus one. And on the denominator, well we've got five times one, so that's five, and we've got this X, so it's five X. And we just quickly check to see whether we have cancelled everything that can be cancelled, and indeed we have. There are no more common factors between the numerator and denominator. So that's the final answer.

Now just one thing to notice is that if you had wanted to you could have cancelled the factors at this stage before you even did the multiplication. You could have noticed it when you do the multiplication you're going to get a common factor of X on the numerator and on the denominator, and you could have cancelled them at that stage, and you could have done similarly for the Y.

Unit 9, Example 14: Dividing algebraic fractions

In this question, we're given two expressions, and we have to write them as single algebraic fractions.

So here's the first one: A over two X, divided by B over three Y. Well this expression is a division of one algebraic fraction, A over two X, divided by B over three Y. Well dividing by a fraction is the same as multiplying by the reciprocal of the fraction. So this is equal to A divided by two X multiplied by the reciprocal of B over three Y, and the reciprocal of B over

three Y is obtained by turning the fraction upside-down, so this is three Y divided by B. So now we have to multiply these two algebraic fractions, and we do that by multiplying the numerators, so that's three AY, and multiplying the denominators, so that's two BX. Two X times B; two BX. And I've written the letters in that order because it's usual in products to write the letters in alphabetic order, unless there's some reason why you shouldn't.

So that's our first example, here's our answer. You should usually check whether there's any further simplification that can take place but in this case there isn't. There are no factors of the numerator and denominator that you can cancel.

So let's look at our next question. We have three X over five Y, divided by Y times Y plus one over X squared, and that's equal to, well dividing by a fraction is the same as multiplying by the reciprocal of the fraction, so this is three X over five Y multiplied by the reciprocal of this, you turn it upside-down, you get X squared in the numerator and Y, times Y plus one in the denominator. And now we multiply the numerators. So this is three X times X squared, three X cubed. Five Y times Y times Y plus one, well, five Y times Y is five Y squared, times Y plus one. And again we look at our final answer and we check to see if there's any cancellation that can take place, and there isn't.

By the way at the beginning of this you might have been tempted to do some cancellation. You see the Y here and the Y up there, you might have been tempted to cancel those. But that would have been a mistake because this is a division and you should wait until after you've performed the division and replaced it by a multiplication before you think about cancelling in that kind of way.

Unit 9, Example 17: Changing the subject

In this question, we're given an equation that involves three variables, F, U and V, and we're asked to make U the subject of the equation. So let's begin by writing down the equation.

So the equation is: it's one over F equals one over U plus one over V. Now you've seen the strategy for making the variable a subject of an equation, and the first step of the strategy is to clear any fractions in the equation. Now this equation has got three fractions: one over F, one over U and one over V. To clear the first fraction we need to multiply both sides of the equation by F, to clear the second fraction we need to multiply both sides by U and to clear the third fraction we need to multiply both sides by V. And the easiest thing to do is just to do those three things all in one step; in other words, to multiply both sides of the equation by FUV, the product FUV.

So let's do that. Multiply by FUV. Well, if you multiply the left hand side by FUV you get FUV over F, and if you multiply the right hand side, where we've got to multiply the whole of the right hand side, and if you think about it, we're going to have this, FUV times one over U plus one over V in brackets, and then when we multiply out what's going to happen to be that each term will be multiplied by FUV. So we're going to get FUV over U

plus FUV over V. And the next step is to simplify that. So we've got FUV divided by F, which will give us just UV. We've got FUV divided by U which gives FV, and we've got FUV divided by V, which gives FU.

Right, so now we've got an equation that has no fractions and no brackets, so we're able to go on to the next stage of the strategy. And the next stage is to look at the terms and to get all of the terms involving the required subject which is U on one side and all the terms that don't contain the required subject on the other side. So let's look at the terms that we've got here. This one is UV, and it's got the required subject U in it. This one, FV, doesn't have the U in it, and this one does have the U in it. So let me get all the terms that have the required subject U on the left hand side and all the other terms on the right hand side.

So let's think about what we need to do. Well, this term has got U in it, so it's going to stay on the left hand side. This term doesn't have U in it, so we do want that on the right hand side so that can stay there, and this term does have U in it, so we don't want it on the right hand side, we want it on the left hand side. So to get rid of this term on the right hand side, we've got to subtract FU. So let's do that. So we subtract FU from both sides. So on the left hand side we've got UV minus FU and on the right hand side where we've still got this FV, and we subtract FU to cancel out that term, so that's all we've got on the right hand side.

So let's just check that we got what we wanted. We wanted all the terms with the required subject U in them to be on the left, and we do have that. This term's got U and this term's got U and there's no U over here, and we wanted all the other terms, all the terms that don't have U in them, to be on the right, and we do have that. This is the only term that doesn't have U in it, and it's on the right. So now we can go into the next step of the strategy.

Well, we can see that what we've got on the left side here is two terms involving U, and that means that we've got to take out U as a common factor. So let's do that. So we take out U. What we're left with in the first term is the V, and what we're left with in the second term is the minus F, and then on the right hand side we've got FV as before. And now we're nearly done because all we have to do is divide both sides by the expression that multiplies the required subject U, and that expression is V minus F, so let's divide by V minus F.

So if I divide the left hand side by V minus F then the V minus F's will cancel, which is what we wanted, and I'll just be left with U, and if I divide the right hand side by V minus F then I'll get FV over V minus F. And that concludes the strategy. So I have now rearranged the equation so that the subject of the equation is U, and we can check that by making sure that U appears by itself on one side of the equation, which it does, and by checking that U doesn't appear at all on the other side of the equation, and you can see that it doesn't, so this is an equation whose subject is U.

And there's one more thing to notice about the working here, and that's that I divided by V minus F. Now you can't divide by zero, so that means that this working isn't valid when V minus F is zero. So the equation that I ended up with isn't valid when V minus F is zero; in other words, it's not valid when V is equal to F.

Unit 9, Example 18: Rearranging an equation with powers

In this question, we're given a formula involving the two variables V and R, and we're asked to make R the subject of the formula. That means that we want to get R by itself on the left-hand side of the equation. So our formula is V equals four thirds pi R cubed. And pi is a constant; its value is about three point one four.

Okay, well you've been given a strategy for doing this, and the first step in the strategy is to clear the fractions. So we have this fraction four over three. So we can get rid of that if we multiply it by three. So we're going to multiply both sides of this equation by three, and we get three V is equal to, well three times four over three is four pi R cubed. So that's the first stage, to clear the fractions.

Well we're trying to get R by itself, so it'll be a good step if we can get R cubed by itself, and we can do that if we divide both sides of the equation by this constant four pi. So let's divide by four pi. Where on the left we get three V divided by four pi, and on the right we get, well if we divide four pi R cubed by four pi we just get R cubed. So now we have R cubed by itself. We want R by itself on the left-hand side, so let's swap the sides of the equation. So swap the sides and just get R cubed equals three V over four pi.

So now we're nearly there. We have R cubed by itself on the left-hand side but we'd like R. Well what we want to do now is perform the operation that reverses the effect of that cube, and the operation that reverses the effect of that cube is taking the one-third power. So we're going to raise both sides to the power one-third, and that's because if we take R cubed and raise it to the one-third power then we get R to the power three times a third, which is R to the power one. That's just R. Okay, so that's our calculation over there. So let's do that. If we raise the left-hand side to the power one-third we get R and on the right-hand side we get three V over four pi all to the power one-third.

So now we've completed our task because on the left-hand side we've have R all by itself and on the right-hand side R doesn't appear.

Unit 10, Example 2: Sketching the graph of a quadratic function

This question is about sketching a parabola, which is the graph of a quadratic equation like this one here. In Parts A and B we're asked to find some information about the parabola, and then in Part C we'll use that information to sketch it.

So let's first look at Part A, and this asked us to state whether the parabola is u- shaped or n-shaped and to find its intercepts. Well, u-shaped or n-shaped, remember that a parabola can have either of two shapes, it can look like this, a u-shape, or this, an n-shape, and which one it is depends on the coefficient of x squared. And there's nice way to remember how that goes. If the coefficient of x squared is positive, then the graph will be this shape;

it looks like a smile, a positive expression. And if the coefficient of x squared is negative, then it's this shape, which looks like a frown, a negative expression. So our particular quadratic expression over here has a negative coefficient for x squared, so it will be this shape, a n-shape. So let's write that down. The coefficient of x squared is negative, so the parabola is n-shaped.

Right, let's look at the second bit of Part A. So we're asked to find the intercepts of the parabola. The intercepts of a graph are the values where it crosses the axes. The x-intercepts are where it crosses the x-axis and the y-intercepts are the values where it crosses the y-axis. So let's begin by thinking about the y-axis. Well, if you think about the shape of a parabola, one of these two shapes, you can see that it will cross the y-axis exactly once. So for example the y-axis might be here, and it would cross the axis exactly once, or the y-axis might be here in this shape, and it would cross the y-axis exactly once. So a parabola that comes from an equation like this has exactly one y-intercept.

Now the points on the y-axis are the points that have x coordinate zero. So to find the y-intercept what we need to do is to substitute x equal to zero into the equation of the parabola and find the corresponding y value. Now you can see that when we substitute x equal to zero into this equation, what will happen to be that the first two terms will be zero and we'll be left with just y equals eight. So in fact the y-intercept turns out just to be the constant term in the equation of the parabola, and that will always happen. Another thing to remember about intercepts is that they are just values, they're not points. So for example the y-intercept that we've found is just the value eight; it's not a pair of coordinates. Right, let's write down what we've found. So we put x equal to zero, and we found that when we substituted that in we got y equals eight. So the y-intercept is eight.

Right, let's now think about the x-intercepts. Well, the x-intercepts are where the parabola crosses the x-axis, and if you think about it there's several possibilities for what could happen. For example, let's think about this shape of parabola. If the x-axis was right through the middle of the parabola then you would have two x-intercepts, if it was right through the vertex of the parabola then you would have just one x-intercept, and if it was off the bottom of the parabola then you would have no x-intercepts. Similarly, for this shape over here, if the x-axis was right off the top of the parabola you'd have no x-intercepts, if it was straight through the top, straight through the vertex, then you would have one x-intercept, and if it went through the middle you'd have two x-intercepts.

Right, so let's look at our particular one here. Well, the x-intercepts are the values where the graph crosses the x-axis, and the x-axis consists of all the points for which y equals zero. So to find the x-intercepts we've got to substitute y equals zero in the equation of the parabola. So let's do that. Putting y equals to zero gives, well, we'll have zero over here, and this expression on the right hand side. So to find the x-intercepts, we've got to solve this quadratic equation. Well, I'm going to write it with the quadratic expression on the left hand side, and I'm also going to multiply both sides by minus one so that I end up with a positive coefficient here because it's always easier to deal with a quadratic equation if the coefficient of x squared

is positive. So let me do that. So I'm going to multiply both sides by negative one, which will give me this, and I'm going to swap the sides. So I've got that.

So the next step is to solve this quadratic equation, and in unit nine you saw a method for doing that, and that is to factorise. Okay, well the coefficient of x squared is one, so that means it's one of the easier factorisations to do. Okay, it's going to be something like this. We're going to have two brackets, each of which has got an x here, and we've got to find the number that goes in here, and it's going to be equal to zero. Now of course not every quadratic expression will factorise, but we're hoping that this one does.

Right, well the two numbers that go in here, as you've seen, they must be such that their product is minus eight and their sum is minus two, and a little bit of thought shows you that the correct numbers are plus two and minus four, and you can just check that by doing a quick mental check by multiplying out mentally. Okay, if you multiply out you're going to get x squared and in the middle you're going to get plus two x minus four x, which all together is minus two x, which is what we want, and at the end we're going to get plus two times minus four which is minus eight, so this factorisation does seem to be correct.

Now what we've got here is the product of two numbers equal to zero. This number here times this number here is equal to zero. So that means that one of the numbers must be zero. So either x plus two is zero or x minus four is zero. Okay, and now all we have to do is to solve these two simple linear equations. The first one gives us x equals minus two and the second one gives us x equals four. So we've now found the two x-intercepts. Let's write them down. The x-intercepts are minus two and four.

Now let's look at Part B, and this asks us to find the axis of symmetry and the vertex of the parabola, and what that means is that we have to find the equation of the axis of symmetry and the coordinates of the vertex. Now let's think about the shape of our parabola. So we know it's at an n-shape. So let me just draw roughly what that looks like over here. Okay. And we want to find the axis of symmetry. Now we can do that by thinking about the x-intercepts. We know that the x-intercepts are the values where the parabola crosses the x-axis, and our particular parabola here has got two x-intercepts, so we know that the x-axis does go through the middle of it, something like this.

So this x-intercept will be minus two and this one will be four. And the parabola is symmetrical. So the axis of symmetry goes down the middle like this. So it's exactly halfway between the two x-intercepts. Now the distance from minus two to four is six. So the distance from minus two to the axis of symmetry will be three. So, in other words, this number here is three bigger than minus two, so it must be one. Or alternatively you could have said it's three smaller than four, which again would give one. So the axis of symmetry is the line that goes through the x value one. In other words, it's the line with the equation x equal to one.

In this case, it was easy to see what the value halfway between the two x-intercepts was, but sometimes it's not quite so easy; for example, if the x-

intercepts were fractions rather than whole numbers. But there's always a way that you can work it out, and that is just to calculate the average or mean of the two x-intercepts; the value in the middle is always just the mean of the two x-intercepts. So let me write that down over here. So the value halfway between the x-intercepts is, it’s the mean of minus two and four, so that's minus two plus four divided by two. So that's two divided by two, which is one, and so the equation of the axis of symmetry is x equals one.

Right so we’ve found the equation of the axis of symmetry, and the next thing we’re asked to do is to find the vertex of the parabola. Well, as you know, the vertex of the parabola is this point here, the point right at the top of the parabola, or it would be right at the bottom if it was a u-shape parabola, and the vertex actually lies on the axis of symmetry. So we already know what its x coordinate is, it’s just going to be one. So let's write that down. So the x coordinate of the vertex is one, and we just need to find its y coordinate. And to do that all we need to do is to substitute x equal to one into this equation, so let's do that.

So substituting x equals one into the equation of the parabola gives, that would be y equals minus x squared, so that's minus one squared, plus two x, so that would be plus two times one, plus eight, and now we simplify this. And you have to be a bit careful here because you notice that this doesn’t mean minus one all squared, it just means minus one squared. So we evaluate the one squared to get one, and the minus is still there, so we get minus one, and then we’ve got plus two times one, so that would be plus two, and the plus eight gives us that. So we’ve got minus one plus two which is one plus eight will give us nine. So the y coordinates of the vertex is nine and so the vertex we now know is x coordinate one, y coordinate nine.

So now we’ve got all the information that we were asked to find and we’re ready to sketch the parabola. Here’s a new page for that. So I’m going to draw some axes. So first of all I’ll draw the y-axis using a ruler. Okay and the x-axis. Right and I know that the y-intercept is eight, so that will be say here, and the x-intercepts are minus two and four, so let me mark the one that's four first. So that will be along the x-axis, half the distance that this y intercept is up to the y-axis, so somewhere like here. Okay and the other x intercept is minus two so that would be half this far from the origin as this one is, so something like this. Okay and let me also put arrows and labels on the axis.

Right and I also know that the axis of symmetry goes through the point, well goes through the value one on the x-axis. So this will be along here about half this distance here but on this side. So it’ll be somewhere like this. So this is the axis of symmetry. Right and let me now try to draw a nice smooth parabola that goes through these three points here and that has this line as its axis of symmetry. So it will be something like this. Right and let me label the values that we used to draw the parabola. So this one was four, this one was minus two, this one was eight, and we know that the vertex is one nine. So that's the sketch that I was asked to produce. And to finish off I’ll just label the parabola with its equation, so it’s y equals minus x squared plus two x plus eight.

Unit 10, Example 5: Using the quadratic formula again

In this question, we're asked to solve the quadratic equation two x squared add four x subtract seven equals zero. So I'll start by writing down the equation for myself. And the first method that I want to consider is factorisation. That's my preference because once you've got used to factorising it is quite often the most straightforward method.

Now if this quadratic expression factorises, we'll be able to form two brackets multiplied by each other still equalling zero, and given that the quadratic expression starts with a two x squared, the first terms in those brackets will be two x and x, and the second terms will come from the factors of negative seven, so the possibilities are plus one minus seven or minus one and plus seven, but if we try those possibilities in the two brackets, you'll soon discover that it's impossible to get the four x term in the original expression. So at that point we can conclude that we can't easily factorise this expression and it's best to move to the method of using the quadratic formula. So that's what we'll do now.

So let's begin again by writing the quadratic formula, x is equal to negative b plus or minus the square root of b squared subtract four a c, all over two a. Where a, b and c are the coefficients of the terms in the quadratic expression. So here a equals two, b equals four and c equals negative seven. So now we can put those values into the quadratic formula to give us x equals minus b, well b is four, so that's negative four, plus or minus the square root, big long square root, four b squared, four squared, subtract four multiplied by a, which is two, multiplied by c, which is minus seven, and then all of that is divided by two times a, which is two.

So now we start to do some of the calculations in there, negative four plus or minus the square root of four squared is sixteen, and then we have negative four times two, which is negative eight, multiplied by negative seven, seven eights are fifty-six, and with the two negatives we'll have plus fifty six, all divided by four on the bottom. And then we can tidy that up a little bit more, negative four plus or minus the square root of sixteen add fifty-six is seventy-two, all over four. So at this point I observe that seventy-two is equal to the product of thirty-six and two, and of course thirty-six is a square number. So I can write the square root of seventy-two will be equal to the square root of thirty-six multiplied by the square root of two, which is equal to six root two, and this was covered in unit three. We can now put this value into the formula so that we've got x equal to negative four, plus or minus six root two, all divided by four.

Now at this stage I want to do some cancelling, but I find it easier to do that if I first split up this expression into two separate fractions. So I can rewrite our expression as negative four over four plus or minus six root two over four, and then the left hand fraction will cancel to minus four over four is minus one, plus or minus, and in the right hand fraction the six and four will cancel to three over two multiplied by the root two. So finally we just need to put that back into the context of the question by writing so the solutions are x equals negative one add three over two root two and x equals negative one subtract three over two root two. So that's solved that particular

quadratic equation, but now I'd like to show you another couple of examples where it's helpful to do a little bit of simplifying before the solving.

Let's first look at solving the quadratic equation x squared add two x subtract seven over two equal to zero, and of course this time the quadratic expression involves a fraction, and my preference is certainly always to work with whole numbers where possible. But because we have an equation here we can multiply both sides by two and hence remove that fraction and still have an equivalent equation to solve. So that's what I'll do. I multiply the left hand side, the whole of the left hand side by two, so I put the expression in brackets, and multiply the right hand side by two, well zero multiplied by two will just be zero, and then now I multiply out the brackets on the left hand side to get two x squared add four x and two multiplied by minus seven over two, the twos cancel to leave me with negative seven, equal to zero. And that's equation we had in the original question, so at this point we would just proceed as before.

Now consider the quadratic equation: negative two x squared subtract four x add seven equals zero. This time we have a negative coefficient of x squared, and it's easier either using the factorisation method or the quadratic formula to have a positive coefficient of x squared, but again because we have an equation here we can do the same to both sides by multiplying by negative one and thus get a positive coefficient for x squared, so that's what I'll do. Negative one multiplied by the left hand side will be equal to zero multiplied by negative one, which is just zero, and then we multiply out the brackets, negative one times minus two x squared gives me plus two x squared, or just two x squared, negative one multiplied by minus four x plus four x, negative one times seven, negative seven. And again we have the quadratic equation that we had in the original question, so we could proceed from this point.

Unit 10, Example 8: Completing the square for quadratics of the form $x^2 + bx + c$

In this question, we're asked to write two quadratic expressions in completed square form, and that means we want to write them in the following form: x plus a number, all squared, plus another number. And these numbers could be positive or negative, so this might be x plus two all squared or x minus three all squared and this could be plus five or minus three. And it's often a good idea to write quadratic expressions in this form because it can give you useful information about the quadratic. For example, it tells you where the vertex of the graph of the quadratic is, and this method also gives you a way of solving quadratic equations. So this can be a useful way of writing the quadratic.

Okay, so here's my first quadratic expression, x squared plus eight x plus ten, and to write it in this form I've got to start with x plus or minus a number, all squared. So I have to decide what this number is. When I am completing the square on this expression, x squared plus eight x, and the number I put in here I've got to choose in such a way that I get an eight x here - the coefficient of x should be eight. And in general you find that the way to do that is to choose the number here to be half of the coefficient of

x. So that's half of plus eight, half of plus eight is plus four, so I'm going to write plus four here.

Well okay let me see what I get if I expand x plus four all squared. So that's x plus four times x plus four, and that's equal to x squared, plus four x plus four x is plus eight x, plus four times four is sixteen. So I do indeed get x squared plus eight x, which is what I wanted, but I also get plus sixteen from this. And in order to deal with that I'm going to write minus sixteen, and the minus sixteen will cancel this plus sixteen, the extra plus sixteen. But now I mustn't forget that I have a plus ten here as well, so I must add that in, plus ten.

Okay now let me simplify that a little bit. Collect those constant terms, x plus four, all squared, minus sixteen plus ten is minus six. So that's my answer, I've now got it in the form x plus a number all squared plus a number, in this case minus six. Well now you should if possible check your working, so let's do that by multiplying out this expression: x plus four squared minus six is equal to. Well I worked out x plus four all squared up here, it's x squared plus eight x plus sixteen minus six, and that's equal to x squared plus eight x plus ten, and that's what I started with. So the answer's correct in that case.

Well let's now do the same thing with this expression: x squared minus three x plus five. Well again I want to write it in the form x plus a number all squared plus another number. So I'll start by writing x and then squared, and I have to work out what this number is, and remember the way to do that is to look at the coefficient of x over here, that's minus three, and take half the coefficient of that. Half the coefficient of x, and that's minus three over two. Well okay what happens if we multiply out that, square this expression, x minus three over two times x minus three over two. Well I'm going to get that equal to x squared. I've got a minus three over two, x minus three over two x, gives you minus three x, and then minus three over two times minus three over two gives you plus nine over four.

Okay so x squared minus three x is what I wanted, but now I've got this extra plus nine over four so I must subtract nine over four. And notice that what I'm subtracting here is this number three over two all squared. If you look back at the previous example you'll see that the sixteen there was this number all squared, and that always happens when you're completing the square. The number that comes in here, well, you have to subtract off the square of that afterwards. Well going back to the second quadratic expression, I mustn't forget that I have plus five on the left, so let me add that on. Okay so this is equal to x minus three over two, all squared. Now I have to collect these terms together, well that's minus nine over four plus, well putting that over the denominator of four, we have twenty, and collecting those together x minus three over two, all squared, plus eleven over four, and that's the answer.

Well let me do a check. Well x minus three over two, all squared, plus eleven over four, well that's equal to, well I did X minus three over two up here within my working. So that's equal to x squared minus three x plus nine over four, plus eleven over four, well now if you combine those fractions together you get this is x squared minus three x, nine plus eleven is twenty, divided by four is five, and that's correct.

Unit 10, Example 10: Completing the square for quadratics of the form $ax^2 + bx + c$

In this example, we’re asked to write two quadratic expressions in completed square form. The quadratic expressions in this example aren’t as straightforward to deal with as the ones you saw in the previous tutorial clip because for these the coefficient of x squared isn’t one, it’s two, in Part A, and it’s minus one in Part B. And there's nothing that you can do to make the coefficient one. Like, for example, you can’t just divide through by two here because we don’t have equations here, we’ve got expressions, and if you were to divide through by two here you would change the expression into a different expression.

So let's look at how you can deal with these quadratic expressions. First of all, let's think about what it means to write these quadratic expressions in completed square form. Well, you saw in the previous tutorial clip that you could write the quadratic expressions there in this form. You could write them in the form x plus a number, all squared, plus another number, and you saw that these two numbers here could be either positive or negative. So for example this could be x plus three, all squared, or it could be x minus five, all squared, and here we could have minus two or we could have plus three.

Now you can’t write expressions like this in quite this form because the coefficient of x squared is not one, but you can write them in something very close. You can write them in this form, a number times, x plus a number all squared, plus another number, and this is what we call the completed squared form of a general quadratic expression. So let's try and write the two expressions here in this form, and here’s how you can do that. Well, as before you look at the first two terms, which in this case is two x squared, plus eight x. Right, let me write down the quadratic expression before I go any further, and that's this, and as I said I’m going to look at the first two terms.

Now I’m going to rewrite these first two terms in a way that gives me a quadratic expression in which the coefficient of x squared is one, and I’m going to do that by taking the coefficient of x squared out of these two terms as a common factor. The coefficient of x squared is two, so I’m going to take that out as a common factor, just out of the first two terms, and that will give me two times x squared. When I take two out of plus eight x, I’m left with plus four x, okay, and I close the brackets, and then I’ve still got this minus seven at the end.

Right, and now what I’ve got here is a quadratic expression in which the coefficient of x squared is just one, and I know how to deal with that because that's what you saw in the previous tutorial clip. Right, so let's now complete the square on this quadratic expression, and I’m just going to focus on this quadratic expression and complete the square on that and I’m going to leave the rest of the quadratic expression completely unchanged at the moment. So to emphasise that let me just write down the brackets that will go round the completed square form there and put in the other bits of the expression.

Right, so in here we’ve got x squared plus four x, and you’ve seen that the way to complete the square on this is to look at the coefficient of x, which is

plus four, and to take half of it. Half of it is two, is plus two. So to complete the square we're going to write x plus two squared. Okay, and let's think about how what we've got here relates to what we've got up here. Well, when we multiply out x plus two squared, we're going to get x squared, which will give us this term, and then the inner and outer terms will give us plus four x, which will give us this, and then multiplying the last terms together we're going to get an extra plus two squared.

So to make this expression in here equivalent to this one up here I've got to subtract the two squared again. Okay, so in other words x plus two squared is the same as this expression up here except that it's got an extra two squared, so to make it the same as this expression up here I've got to subtract the extra two squared again. Right, now before we go on let's just slightly simplify the expression in the brackets by writing two squared as four. Okay, what you need to remember is that the extra term that you have to subtract is always the square of the term that you've got in here.

Now, to get towards the expression that we want, the type of expression that we want over here, the next step is to multiply out the outer brackets here only. So I'm going to multiply this two into what's inside the outer brackets. So that will give me two times the first term, which is two times x plus two, all squared, and then we're going to have two times the next term, which is minus four, two times minus four is minus eight, so I'm going to have minus eight, and then I've still got this minus seven at the end. And now you can see that I'm much closer to the expression that I wanted, the type of expression that I wanted, all I need to do is to collect these two constant terms at the end, so let's do that. So that's two, x plus two, all squared, minus eight minus seven is minus fifteen. So I have written my quadratic expression in the form that I wanted. It's a number times x plus two all squared minus a number.

Now when you've done an algebraic manipulation like this, especially one which is really quite complicated, it's a good idea to check your work, so let's do that. Okay, and you can always check a completing the square type manipulation by just multiplying out again. So let's take the expression that we ended up with down here and let's multiply out, there's two, x plus two squared, minus fifteen. Right so we multiply out these brackets here, well to help me do that let's write them like this, x plus two squared is x plus two times x plus two, and then we've got the minus fifteen, so that's two. And then we multiply out these brackets. Now what we get when we multiply out these brackets is still all going to have to be multiplied by the two. So I still want to enclose it in brackets to make it clear that the two multiplies the whole thing.

So we're going to get x times x, which is x squared, in the outer and inner term, so that'll be plus two x, and plus two x, which will give plus four x, and then we're going to have plus two times plus two, which will give plus four, and we've still got the minus fifteen on the end, and then we multiply the two into the brackets, so that will give two times x squared, so that's two x squared, and now we've got two times plus four x, which will give plus eight x, and then we've got two times plus four, which will give plus eight, and we've still got the minus fifteen. And then to simplify this all we have to do is to collect the two constant terms at the end. So we've got two x

squared plus eight x, plus eight minus fifteen, so that's minus seven, and let's now check to see whether that is what we started with, and indeed it is. So it looks like our completing this squared manipulation was correct.

Now let's look at Part B. In this case the coefficient of x squared is minus one. But I’m going to deal with this quadratic expression in a very similar way to the way that I dealt with the one up here. So let's begin by writing it out. So it’s minus x squared plus eight x minus seven, and what I do is to look at the first two terms, so it’s minus x squared plus eight x, and I’m going to take out the minus sign here so that I end up with in the brackets an expression in which the coefficient of x squared is just one. Okay so I’m going to take the minus sign out, but just out of the first two terms, that will give me minus x squared minus eight x - because remember taking out a minus sign just means changing the signs of everything that's inside the brackets - and I’ve still got the minus seven at the end.

Right, the next thing to do is to focus on the expression inside the brackets and to complete the square on this expression. So let's just carry down everything else in the expression to the line below. Okay, everything else stays exactly as it is and I’m just going to focus on the expression in the brackets. Now as you’ve seen the way to complete the square on an expression like this is to look at the coefficient of x, and it’s minus eight, and then you take half of that for the constant that you’ve got to put inside the brackets here. In other words, we’re going to have x inside the brackets squared, and the number that's got to go here has got to be half of minus eight, so it’s got to be minus four.

Now, as you know, when we multiply out these brackets here, we will definitely get x squared, and we will definitely get minus eight x, but we will also get a constant term, and the constant term will be minus four squared. And so if we want this expression inside these brackets to be exactly the same as the expression up here we’ve got to subtract the minus four squared again. Well subtracting minus four squared is just the same as subtracting four squared, so let me just write it like this, minus four squared. So I’ve now got an expression which has got this thing in it, it’s got x minus four squared, and that's going to be this part here of the expression that I want to end up with. So I don’t want to do anything to this bit of the expression, but I do want to simplify the rest of it. So let's do that.

So the first thing to do is just to write this four squared here is sixteen. So we’re going to have minus x minus four, all squared, minus sixteen, and we’re going to have the minus seven at the end, and the next step is to take this minus sign inside the outer brackets, but remember I don’t want to do anything to these inner brackets because I want to leave this bit of the expression just as it is. So taking the minus inside the outer brackets is going to give me minus x minus four, all squared, and then minus minus sixteen will be plus sixteen, and I’m still going to have the minus seven at the end, and then all I need to do now is to simplify this bit at the end by collecting these two constants. So I’m going to have minus x minus four, all squared, plus sixteen minus seven is plus nine. And now I do have an expression which is of this form up here. It’s a number, which in this case is just minus one, times x plus a number, all squared, which is x minus four,

all squared, plus a number, which in this case is plus nine. So I have written it in completed square form.

Now as for Part A, it's a good idea to check that our working was correct. So let's do that. So check, I take the expression that I ended up with the completed square expression. So that's minus x minus four, all squared, plus nine, and I'm going to multiply out and check that I do get the expression that I started with. Well, so that will be minus x minus four, all squared, that just means x minus four times x minus four, and then I've got the plus nine, right, and then I'm going to have minus, and when I multiply out the x minus four and x minus four, what I end up with has all got this minus sign in front of it, so I'm going to keep it in brackets so that I remember to multiply it all by the minus sign.

So let's multiply that out. So x times x gives x squared, looking at the outer and inner terms I'm going to have been minus four x and minus four x, so all together that will be minus eight x, and at the end I'm going to have minus four times minus four, which is plus sixteen, and then I've still got the plus nine at the end. So now I'm going to take this minus sign into the brackets, and that means changing the sign of all the terms inside the brackets. So that will give me minus x squared, plus eight x, minus sixteen, and I've still got the plus nine at the end. Now I just need to collect together the two constant terms at the end. So I'm going to get minus x squared, plus eight x, minus sixteen plus nine, which is minus seven, and now the expression that I've ended up with here is indeed the expression that I started with over here, so it looks like the completing this square that I did was correct.

Unit 10, Example 13: Maximising an area

In this problem, we're asked to maximise the area of an enclosure that's going to be bounded by a fence going around four sides but also in one corner by a barn. And we're given the dimensions of the barn, eight metres by twelve metres, and we're also told what the total length of fencing is, one hundred metres.

Well the first step here would be to introduce some symbols to denote the length and width of the enclosure. Let's suppose that the length is denoted by X metres and the width by Y metres, that's the total length and the total width. In terms of those, I can write down expressions for the lengths of fence which run from the barn. This length here will be Y minus eight and this length here will be X minus twelve.

Now what I'm after is the area of the enclosure. The area of this L-shaped enclosure will be equal to the area of the large rectangle minus the area covered by the barn. But area of the large rectangle is the length times the width XY, and the area covered by the barn which has to be subtracted off is eight by twelve, and that's XY minus ninety-six.

So we're looking at possible configurations in which the X and Y take different values, and correspondingly the area takes different values, and we want to maximise the value of the area. So we want to find the maximum value of XY minus ninety-six. Well at the moment we can't make progress

directly with that, but there's one other piece of information that we haven't yet put into mathematical form, and that's about the total length of the fencing, one hundred metres.

The total length of the fencing in metres is one hundred, and how is the fencing made up? Well, starting from the left-hand side of the barn we have a section of length X minus twelve, then the entire width, Y, the whole length, X, and the remaining bit back to the barn, Y minus eight. That's, simplifying that I have an X and an X making two X, a Y and a Y making two Y, minus twelve and minus eight gives minus twenty.

So one hundred equals two X plus two Y minus twenty. If we add twenty to both sides that becomes one hundred and twenty equals two X plus two Y, or dividing through by two and swapping sides of the equation, X plus Y equals sixty. If X plus Y is equal to sixty that means I can express Y in terms of X, subtract Y from both sides, that gives Y equals sixty minus X. Well now I can take this expression for Y in terms of X and fit it back into my expression for the area. That'll give me area equals XY where Y is now sixty minus X, X times sixty minus X, and then the minus ninety six is still there. Multiplying out the bracket, that is minus X squared plus sixty X minus ninety-six.

So now I have an expression for the area in terms of one variable, X, the length of the enclosure. In order to make further progress, the next step is going to be to complete the square on the minus X squared plus sixty X. So to do that I will first take out a minus and write this is as minus X squared minus sixty X, all in a bracket, minus ninety six. And now I can complete the square on this to get minus bracket. Complete the square on X squared minus sixty X, that's X minus thirty, all squared, minus nine hundred, and this thirty comes about because it's a half of the sixty that we had up here, and the nine hundred is thirty squared, and then not forgetting the minus ninety six which still comes down here. That's the same as minus X minus thirty squared, and then I have minus of minus nine hundred giving plus nine hundred, minus ninety six, so I have plus eight hundred and four.

Well looking at this expression I can see this is a square here. A square can never have a value less than zero. So minus a square can never have a value greater than zero. So in fact the eight hundred and four sitting here must be the maximum possible value of the area of the enclosure. So eight hundred and four is the maximum area, and since the linear dimensions here are in metres, the area is measured in square metres, metres squared.

Well it's also of interest of course to see how this maximum area is achieved, what are the length and width concerned. It's got to be achieved by this bracket being zero and that corresponds to X equals thirty. So this maximum area occurs when the length X is thirty and the linear dimensions are in metres. And what's more, if we put X equals thirty back here in our equation for Y, we can see that Y too is thirty, so the width is thirty metres.

So the maximum area is obtained when the rectangle surrounding the barn is actually a square, and here's the maximum area, eight hundred and four square metres.

Unit 12, Example 1: Finding unknown lengths

In this question, we’re asked to find the length of an unknown side in each of two triangles. In Part A, we’re looking at this triangle here, and my preference in such questions is always to draw the triangle for myself so that I can start to absorb the information that I’m given and what it is I need to work out. So here’s the triangle, and we’re told that there's a right angle in this corner, and we’re told that this angle is of twenty-five degrees, this side has length nine, and this side which we want to find has length x.

So in this triangle the side opposite the right angle, that's the hypotenuse, is of length nine, and the side opposite the known angle of twenty-five degrees is x, so that's the opposite side. So in order to relate these three I need to use the trigonometric ratio which relates an angle with its opposite side and the hypotenuse, and that means using the sine ratio. Sine of theta equals the length of the opposite side over the length of the hypotenuse.

So now I’ll put in the information that I’ve got for this particular triangle into that. So in this case the sine of twenty-five degrees is equal to the length of the opposite which is x over the length of the hypotenuse which is nine. I’ve now got an equation with an unknown of x, and I need to first rearrange this so that x becomes the subject of the equation, and to do that I want to multiply both sides by nine. So on the left hand side we’ll have nine multiplied by sine twenty-five degrees equals, and on the right hand side x over nine multiplied by nine is just x, and then I’ll just rewrite that with the subject x on the left hand side.

We’re now ready to carry out a calculation, and for this I’ll need to use my calculator, and the first thing to do is to check that my calculator is currently working in degree mode because our angle is given in degrees, and the simplest way to do that is to key in sine ninety on the calculator, press equals, and if I get the answer one, then it is indeed working in degree mode at the moment. If you get the answer starting nought point eight nine, then it’s in radian mode and you need to change the calculator to be working in degrees.

So having checked that I can now proceed with this calculation. I could do this calculation all in one go on the calculator, and that would be fine, but if you want to take things a little bit more slowly and show some working we can first evaluate sine of twenty-five. So x is now equal to nine multiplied by, and sine twenty-five is nought point four two two six one eight and so on. At this stage, we want to multiply nought point four two two six one eight and so on by the number nine. So whilst nought point four two two six one eight and so on is on the calculator screen, just immediately press multiply by nine, and then you avoid having to rekey in nought point four two two six one eight etc and making any possible errors. So if we do that we get three point eight zero three five six and so on.

So we’ve now completed the calculation, but what we need to do is to relate this result back to the question that we were asked. And in the question we were asked to find the length x, so x will be equal to. We also need to consider whether there’s any units to take into account, which there aren’t in this particular case. Finally, we need to round the answer to three significant figures, as requested in the question. Here the first three significant figures

are three eight and zero, the next figure is a three, so we round the answer down to three point eight zero to three significant figures. Note that we need to include this zero to confirm that the answer is given to three significant figures.

Now we can move on to Part B, and again I'll start by doing a quick sketch of the triangle for myself; the right angle's at the top, this angle is fifty-five degrees, this side has length y and this side has length seven. So in this triangle, the side opposite the right angle, the hypotenuse, is of length y, the one we want to find, and relative to this angle of fifty-five degrees, the side of length seven is adjacent to that angle. So this time I want the trigonometric ratio which relates an angle to its adjacent side and to the hypotenuse, and that means using cos of theta equals adjacent over hypotenuse.

So now let's put the information we have into that. Cos of fifty-five degrees equals the length of the adjacent, which is seven, over the length of the hypotenuse, which is y. So now we have an equation with an unknown y, so we'll rearrange the equation to get y equals on the left hand side. This can be done in one step, but I'll do it in two, by first multiplying both sides by y. So on the left hand side I'll have y multiplied by cos of fifty-five degrees and on the right hand side seven over y multiplied by y is just seven. Next I divide both sides by cos of fifty-five degrees to get y equals seven over cos fifty-five degrees.

Now I want to evaluate this, and again I could do it as one calculation, seven divided by cos of fifty-five degrees, or I can write an intermediate step, seven divided by cos of fifty-five degrees, which is nought point five seven three five seven and so on. Having found that result on my calculator, nought point five seven three five seven and so on, I immediately want to calculate seven divided by that number, and the way to do that, whilst that value is still on your calculator screen, is to key in seven divided by and then use the answer key, and the calculator will interpret that as seven divided by the previous answer that is seven divided by nought point five seven three five seven and so on, and if I do that I get the result twelve point two zero four and so on.

Finally, I need to relate that to the question, so I finish off by writing hence y, and first of all I need to consider if there's any units, which there aren't in this case, and secondly note that I'm asked to give the length to two significant figures, in this case the first two significant figures are the one and the two, the next figure is a two, so I round down to get y equals twelve to two significant figures.

Unit 12, Example 4: Finding unknown angles

In this question, we're asked to find an unknown angle theta in each of these two right angle triangles. So look at the first triangle here. Here the unknown angle is theta, and the first thing I'm going to do is to label the sides of the triangle. Well, the side of the triangle, the longest side that's opposite the right angle is the hypotenuse, and now the side that's opposite the unknown angle I'll label that as opposite, and the side next to the unknown angle, that's the adjacent side. Well, in this question we've been

given the length of the hypotenuse and the length of the opposite side in relation to this unknown angle theta.

So we're going to use the sine ratio because sine of this angle theta is equal to opposite, the length of the opposite side divided by the length of the hypotenuse. Okay, so in this triangle sine theta is equal to the length of the opposite side, that's ten, divided by the length of the hypotenuse, thirty, and that's a third. So sine theta is equal to a third. So we're trying to find an angle theta whose sine is one third, and we do that by using the sine to the minus one operation which reverses sine. So theta is sine to the minus one of one third.

Now you can find sine to the minus one using your calculator, and the way you do that is to first press shift, then the sine key, and then type in one divided by three. And if you do that, you'll get the answer nineteen point four seven one and so on, and that's an answer in degrees. So we'll put the unit in there, nineteen point four seven one degrees. Now let's look back at the question, we're asked to find the angle theta to the nearest degree. So let's write our answer as hence theta equals, well to the nearest degree, this is nineteen degrees to the nearest degree.

Right, so let's look at the second triangle, that's this one. Here the unknown angle is theta, and let's label the sides as we did before. The longest side opposite the right angle is the hypotenuse, the side opposite theta is the opposite side, and the side next to theta is the adjacent side, and in this case we've been given the length of the opposite side and the length of the adjacent side. So that tells us that we want to use the tangent ration, tan theta is equal to the length of the opposite side divided by the length of the adjacent side. So in our triangle tan theta is equal to ten divided by thirty, a third.

So we need to find the angle theta whose tangent is one third, and we do that using the operation which reverses the tangent, that's tan to the minus one of one third. Now you could do that on your calculator as well by first pressing the shift button, then the tangent button and then typing in one divided by three, and if you do that you get the answer eighteen point four three four and so on, and that's an answer in degrees, again. Well once again we were asked to find the angle theta to the nearest degree, so let's write that down, hence theta is equal to, well to the nearest degree this is eighteen degrees, to the nearest degree, and that's the answer.

Unit 12, Example 7: Using the Sine Rule without vertex labels

In this question, we're given a triangle, and we're given various pieces of information about the triangle, we're given two of the angles and one of the sides, and we're asked to find another piece of information about the triangle, namely the length of one of the sides which is marked by the letter x here. Now this triangle isn't a right angle triangle, neither of these two angles are ninety degrees, and you can see that if you subtract it, the sum of these angles from a hundred and eighty degrees, you're not going to get ninety degrees, so this angle here are not ninety degrees either. So we can't just simply use one of the trigonometric ratios, sine, cos or tan to work out

the unknown side in this triangle. But what we can use in this case is the sine rule. We can use the sine rule in any triangle if the information that we’re given is sufficient to define the triangle and if the information includes an angle and the side opposite, and we do have that here so we can use the sine rule.

Now let me remind you what the sine rule is. Well it applies to any triangle, so let me just draw a general triangle here, and let me label its vertices, A, B and C, and the normal convention is to label the sides opposites these vertices with the lowercase letter that corresponds to the uppercase letter in the vertex. So, for example, the side opposite the angle at A would be labelled with lowercase a, the side opposite the angle at B would be labelled with lowercase b, and the side opposite the angle at C would be labelled with lowercase c. So this side is length a, this is length b and this is length c, and this is what the sine rule says. It says the a over sine A, okay, in other words, the length a over the sine of the opposite angle is equal to the length b over the sine of the opposite angle, which is equal to the length c over the sine of the opposite angle. The sine rule says that these three ratios are all equal. So this is the sine rule.

Well let's apply the sine rule to the triangle that we’ve got over here. So by the sine rule, okay, and what it tells us is that any length over the sine of the opposite angle will always give the same ratio. So we want to know what this length is, so that length is x, and the sine rule tells us that we look at x over the sine of the opposite angle, which is sine eighty degrees, and that's going to be equal to this length here, ten, over the sine of the opposite angle to that length, which is thirty-five degrees. So it's ten over sine thirty-five degrees. And now what we’ve got here is an equation in the unknown x that we can solve to find it, and the equation of all those various numbers, ten and these two numbers here, sine eighty degrees and sine thirty-five degrees.

Well, to solve this equation, we want to get x by itself on one side, and we can do that by multiplying both sides by sine eighty degrees, so let's do that, and that gives us x equals ten sine eighty degrees over sine thirty-five degrees. So now we’ve got an expression here that we can type into our calculator to find the value of x. Now in my calculator I would do this by typing ten and then pressing the sine button, and that will automatically open a bracket for me, and then I would type eighty, and I would have to close the bracket using the close bracket button on my calculator, and then I could type divides using the divides key, and then I would type sine thirty-five degrees and press the equals key.

An alternative way to do it on my calculator would be to type this expression in exactly as it looks by using the fraction key. So I’d press the fraction key, and then I would type this into the numerator and this into the denominator and press equals. But however you do the calculation you should find the answer is seventeen point one six nine five and so on. Now the question asked for the length to be given to three significant figures. So when I write down my final concluding sentence I’m going to do that rounding. So x equals seventeen point, well three significant figures takes me to here, and the digit after the one is a six, which is greater than five, so

I round it up, so I'm going to get seventeen point two, and that's to three significant figures.

Unit 12, Example 9: Using the Cosine Rule without vertex labels

In this question, we're asked to find the length of the side of a triangle, and we know the lengths of the other two sides of the triangle and the size of the angle between those two sides. We can't assume that either of the other angles is a right angle, and because we don't know the size of an angle and its opposite side we can't use the sine rule. So this means that we need to use the cosine rule.

Now to remind you of the cosine rule, in a triangle, where we have an angle B here, and the side opposite little b, and the other sides labelled a and c, then the size of the side b squared will be equal to the sum of the other two sides, each squared, a squared add c squared, that looks a little bit like Pythagoras's theorem, but then because we haven't got a right angled triangle, we need to subtract two times the product of those two other sides with cos B. So the cosine rule tells us that b squared equals a squared add c squared subtract two a c cos B.

In our triangle the known angle, thirty-five degrees, is opposite the side x. Therefore, by the cosine rule, x squared will be equal to the sum of the squares on the other two sides, that is ten squared add twelve squared, subtract two times the product of the other two sides, multiplied by the cos of that known angle, thirty-five degrees, subtract two times ten times twelve times cos of thirty- five degrees.

Now if you feel you need to take that a little more slowly, you can relate the general cos rule to the particular triangle that you have. So in our case this b here in our triangle is equal to x, a is equal to ten, c is equal to twelve, and the angle B is thirty-five degrees, and then substitute those values into the formula for the cosine rule to get the expression that I got over here.

At this point, you could choose to put this complete calculation into your calculator by keying in ten squared, add twelve squared, subtract two, multiplied by ten, multiplied by twelve, multiplied by cos of thirty-five degrees, and pressing equals should then give you the answer. Or you may prefer to put some intermediate lines of working by first calculating ten squared as a hundred, twelve squared, a hundred and forty-four, subtract, and then two times ten times twelve is two hundred and forty to be multiplied by cos of thirty-five degrees.

At this stage, my inclination would be to put that calculation into the calculator. If you do want to do further intermediate working though, you just need to be careful and remember that you need to multiply this negative two hundred and forty by cos of thirty-five before you try and subtract the two hundred and forty from the hundred plus a hundred and forty-four. If you remember, it's important that the multiplication takes precedence over adding these numbers. Whichever way you do it, you should get the answer forty-seven point four zero three five zero nine and so on.

Well, that's found the value of x squared, but of course we want the value of x, which will be equal to the square root of forty-seven point four zero three five zero nine and so on. Having got the answer of forty seven point four zero three five zero nine and so on, on your calculator screen, to find the value of x you should immediately press the square root button followed by the ands button, and that will calculate the square root of the previous answer, that is the square root of forty-seven point four zero three five zero nine etc. And if you do that, you'll get the value six point eight eight five zero two and so on.

So our answer to the question is that hence the value of x to the required two significant figures will be, well the two significant figures are six and eight, the third figure is eight, so we'll need to round up to get six point nine to two significant figures.

Unit 13, Example 5: Understanding logarithms

This question is all about logarithms. It's about what it means to find the logarithm of a number. Now when you're thinking about logarithms you always have a particular special number in mind, and that number is called the base, and the particular base that we're being asked to look at in this question is base ten. So this means that when you're asked to find the logarithm of a number what you're really being asked is what power do I have to raise ten to to get to that number.

So, for example, let's look at the number a hundred. If you're asked what is the logarithm of a hundred to base ten, then you're being asked what power do you have to raise ten to to get a hundred, what number would have to go here? And the answer is just two. So that means that the logarithm of a hundred to base ten is two, and we write that down like this: log to base ten of a hundred is two. So a logarithm is a power. The logarithm of a hundred to base ten is the power that you've got to raise ten to to get a hundred.

Right, let's look at the particular numbers that we're asked to look at in this question. So Part One is a million. Let's write down a million. That's one followed by six zeros. And if I want to find the logarithm to base ten of a million then I've got to answer the question what number do I have to raise ten to to get a million? What number has to go here? And the answer is just six, because ten to the six is one followed by six zeros. So we found that the log to base ten of a million is six.

Right, let's look at the next part of the question, and we're asked to find the log to base ten of the number one. Right, where again we've got to ask ourselves the question, what number do we have to raise ten to to get one? In other words, what number goes here? And you know that any number to the zero is one, so in particular ten to the zero is one. So this equation tell us that the log to base ten of one is zero. Let's write that down. The log to base ten of one is zero.

Right, let's look at Part Three, so that's this number here. Zero point zero zero zero zero one, a decimal point followed by four zeros and then a one, and again we ask the question what number do I need to raise ten to to get this given number here? And this number is less than one, so the number

that's going to have to go here is going to have to be a negative number. Let's think about what it’s going to be. Well you know that nought point one, that just means a tenth, and you’ve seen that you can write a tenth as ten to the minus one, and nought point nought one you know is one over a hundred, in other words one over ten squared, and you’ve seen that that can be written as ten to the minus two. Let's do one more over here, nought point nought nought one you know is one over a thousand, which is one over ten to the three, which can be written as ten to the minus three.

So for a number like these ones here to see what power needs to go on the ten, well it’s certainly going to be negative and to find which particular number it is you can see that you need to count how many places you need to move the decimal point along until it’s just after one. So you need to move it three places here, and that's why the number is three here, similarly it has to be moved by two places here, and that's why the number is two here. So going back to our number over here, we need to move the decimal point by five places to make it after one, and so the number that we’ve got here is ten to the minus five. Right, and then this equation here just tells us that the log to base ten of our number nought point nought nought nought one is minus five. I found these answers without using my calculator, as the question asked me to do, if you want you could use these numbers to practise finding logs on your calculator.

Now let's look at Parts B and C. I’m going to take a new page to do these parts. The three numbers that we looked at in Part A were all powers of ten where the index was an integer, and that meant that you could find their logarithms to base ten without using a calculator. In general, if you’re asked to find the logarithm to base ten of a number, such as this one here, which isn’t a power of ten in which the index is an integer, then you have to use your calculator. But by thinking about the number you can get a rough idea of how big its log to base ten is going to be, and that can be helpful, so that when you get an answer from your calculator you know whether it seems about right.

So let me show you how that works. I’m going to draw a number line down here so that you can see the kind of things that you need to think about, and I’m going to mark on the number line some powers of ten in which the index is an integer. So for example here I’ve got the number one, which is ten to the zero, and I suppose here I’ve got ten, which is ten to the one, and somewhere along here, in fact it’s ten times as far from ten as ten is from one I’m going to have a hundred, which is ten squared, and somewhere much further along here I’m going to have a thousand, which is ten cubed.

Now we know that the log to base ten of one is zero, the log to base ten of ten is one, the log to base ten of a hundred is two and the log to base ten of a thousand is three. Now let's think about a number that lies between these numbers. So, for example, say a number in here. Well, since the log to base ten of ten is one, and the log to base ten of a hundred is two, the log to base ten of my number in here is going to be between one and two. So that's the kind of thing that you can think about to see roughly how big a log to base ten will be. So, for example, if you look at this number up here, five hundred and sixty eight, it lies somewhere along here. It’s bigger than a

hundred but it's smaller than a thousand, and that means its log to base ten will be between two and three.

Right so let's use this idea to answer the question. The question says find the two consecutive integers between which the number log to the base ten of five hundred and sixty eight lies, and then we're asked to do the same for another number. So we've just done this one. So let's write down what we've found. We said that five hundred and sixty eight is between, well it's between a hundred and a thousand; in other words, it's between ten squared and ten cubed. So the log to base ten of five hundred and sixty eight is between two and three.

Okay now we can do something very similar for this number. It's difficult to see what's going on in this number line here because that number will be somewhere in here, but we can use exactly the same idea. We want to find a power of ten, well we want to find two powers of ten with integer indices between which this number lies. Well, our number is zero point zero three seven. And this number, well it's between zero point zero one and zero point one. Okay it's definitely bigger than zero point zero one and it's definitely smaller than zero point one. Okay. And these two numbers definitely are numbers which can be expressed as powers of ten, in which the powers are integers; it's just that the powers are negative in this case. So zero point zero one, well that is ten to the minus two, and zero point one, well that's a tenth which is ten to the minus one. So the log to base ten of zero point zero three seven is between minus two and minus one.

Right, so let's look at Part C. Let's begin just by doing this to keep this separate. Well in Part C we're asked this, the logarithms to base ten of two numbers are four and minus one, respectively, what are the numbers? Well if the log to base ten of a number is four then that means that the number that you have to raise ten to get the number is four. In other words, it's just saying that the number is ten to the four, which is ten thousand. Let's write that down. If the logarithm to base ten of a number is four then it means that the power that you have to raise ten to get the number is four, so the number is just ten to the four. And the number is ten to the four, which is ten thousand.

Okay, so that's that one. Let's now look at this one, which is minus one. Okay well if the logarithm to base ten of a number is minus one, which is what we're asked here, then the number is, well it just means that the power that you have to raise ten to to get the number is minus one. So it simply means that the number is ten to the minus one, which you know is a tenth. Or, alternatively, you could write it as nought point one, it doesn't matter. So the number is just a tenth. So in Part C we were reversing the process of finding a logarithm that we used in Part A.

Unit 13, Example 11: Solving another exponential equation

This question asks you to find the time it takes for an insect population to go from two hundred to four hundred, given that the population size increases by ten percent each week. So let's talk about what's happening here. We start off with a population of two hundred, so that's at the start,

and the population increases by ten percent each week, and that means that it’s increased to a hundred and ten percent of its initial value after a week. Well, a hundred and ten percent is a hundred and ten divided by a hundred, this is the factor by which it increases, and that's one point one. So each week the population size increases by multiplying it by one point one.

So after a week the population size will be two hundred times one point one, that's after one week. Then after a second week well we multiply by one point one again, two hundred times one point one times one point one, and that's equal to two hundred times one point one squared, and you can see that if we carry on like that, well the next week it’ll be two hundred times one point one cubed and so on. So here the population is modelled by an equation of the following form. The population is modelled by, well we’ll call P the population, and what we have here is two hundred times one point one to the power, and I’m going to call it T, where T is the number of weeks. So T is the number of weeks, so there it was one and two and so on, and P, capital P, is the size of the population after T weeks have passed. Okay, so there's our equation which models this population. What are we asked to do? We’re asked to find out after how many weeks, after how long the population will have reached four hundred.

So here’s our question: if P is equal to four hundred, what is T? Well, okay let's substitute the value P equals four hundred into our equation. So that will give us four hundred equals two hundred times one point one to the power of T, and we have to find T. Well we can do one a simple rearrangement; we can divide both sides by two hundred. So let's do that. Well four hundred divided by two hundred is two, and that gives us one point one to the power of T over here on the right. So that's a little simpler.

But the unknown T here is a power, and I want to rearrange the equation somehow to get T all by itself to make T the subject. Well, because T is a power I’m going to do that by taking logarithms. So I’ll take to log to the base ten of both sides. So log to the base ten of two is equal to log to the base ten of one point one to the power of T, and just to make this absolutely clear let me put brackets around there.

Okay, what do we do now? Well there's a useful rule for dealing with logarithms of this form that you’ve seen in the unit. Let me tell you what that is. It's if you’ve got log to the base ten of something like this, one number to the power of another, then what you can do is take this T and move it to the front. So this is equal to T times log to the base ten of one point one. We don’t need the brackets there anymore. So that's the property I’m going to use here. Okay so that's equal to T times log to the base ten of one point one, and you can see now that T is no longer in the power, no longer a power, but it's, had been freed from the logarithm if you like.

Well we can rearrange this equation in the usual kind of way by dividing both sides by log to the base ten of one point one, this number, and if we do that and swap the sides we get T is equal to log to the base ten of two divided by log to the base ten of one point one, and now that's a number you can work out on your calculator. You could do that by pressing the log button, the log to the base ten button, and inputting two, and then you would close the bracket before using the division sign, then log to the base ten

again of one point one, close the brackets, press the equals sign, and the answer you would get should be seven point two seven two five and so on.

Okay, well that's a rather precise answer there, and we've just been asked how long it will take for the population to increase from two hundred to four hundred, so we'd give a less precise answer than that. Perhaps we would say, hence the population increases from two hundred to four hundred in just over seven weeks, and actually it's about seven weeks, this number is about seven weeks and two days, two days is two sevenths of a week, and you can check that's approximately equal to the point two seven.

Well there's an observation I'd like to make about this particular question. If you go back to this equation here, two equals one point one to the T, or in fact to this equation four hundred equals two hundred times one point one to the T. Supposing the numbers had been not two hundred and four hundred for the initial population and the final population, supposing they'd been three hundred and six hundred, or a hundred and two hundred, then in each of those cases when we cancelled we'd get this equation, two is equal to one point one to the T. The rest of the calculation would then be the same and we'd get this answer of just over seven weeks. In fact this population which increases by ten percent each week will always double in just over seven weeks no matter what the initial starting value is. And one further remark, you might have been wondering why I took logs to the base ten at this point in the calculation, well in fact you could have taken logs to any base, but the reason for taking logs to the base ten is that you have a button for logs to the base ten on your calculator.

Unit 14, Example 5: Using sketch graphs to solve trigonometric equations

In this question, we're asked to find all the solutions of the equation sine theta equals a half between the angles zero degrees and three hundred and sixty degrees. And we're going to do this using the graph of y equals sine theta. So first let's start by drawing that graph. I'm going to use the ruler.

So I have a theta axis here for the angles, so let's mark that theta, and a y axis. There's the origin. And on the theta axis I'm going to mark from zero to three hundred and sixty degrees. So this axis is, the units are degrees, but I'm going to mark those on the numbers. There are various ways of marking units on axes and this is one of them. So I'm going up to three hundred and sixty degrees, and halfway between is a hundred and eighty degrees, and then I'm going to need ninety degrees and two hundred and seventy. And I want to draw on this the graph of y equals sine theta, and I know that the values of sine theta are between one, so let me mark one there, and minus one.

Okay, so what I'm going to do is plot on this graph some values that I know, some well known vales of sine theta, and then draw a small smooth curve through those values. Now I know that sine of zero is equal to zero. Sine of zero degrees is equal to zero so I can plot that point, and I know the other points where it crosses the axes, the theta axis are a hundred and eighty degrees, sine of a hundred and eighty degrees is zero, and three hundred and sixty degrees. So I've got three points on the theta axis. And I

know the sine of ninety degrees is one, so let me plot a point here above that, and sine of two hundred and seventy degrees is minus one, so let me plot that point here.

So now I’ve got five points of the graph plotted, and I’m going to draw a smooth curve through them. So the curve rises from zero here and goes smoothly through that point, and then it decreases and goes through a hundred and eighty degrees - that’s not very accurate but it’ll do - and decreases smoothly and goes through this point, and then increases and goes through that, and of course it extends to the right and the left of zero and three hundred and sixty degrees. So there’s my graph of y equals sine theta, let me label it.

Well, my graph isn’t very neat but the graph of y equals sine theta does have some symmetry properties, which I’m going to point out, and one of them we are going to use. So if you look at the part between zero and a hundred and eighty degrees it’s symmetrical in this vertical line which I’ll draw. So this is the line that passes through ninety degrees. So we can write that as theta equals ninety degrees. So sine theta rises to the left of that giving the same shape as sine theta falls to the right of it, and we’re going to use that property in this solution.

Okay, so we want to find the values of theta such that sine theta is equal to a half. So those will be the heights. We need to find which points theta have height a half. So what I’m going to do is to draw a horizontal line at height a half. So there’s a half and I’m going to use the ruler again. So there’s a horizontal line of height a half. And where y equals sine theta crosses y equals a half those are the values of theta that we’re looking for. So it crosses at these two points. So let’s show where they are on the theta axis. Let’s draw this vertical line. So there are two solutions, and we have to find these two values.

One solution of sine theta equals a half, so one solution. We always find one solution of this equation by using the sine to the minus one button on our calculator. So one solution of that is theta equals sine to the minus one of a half. And if you use your calculator you’ll find that that's equal to thirty degrees. And you may be able to do that without using your calculator because you may remember that sine of thirty degrees is equal to a half - that’s one of the standard values that is worth remembering.

So we know that one solution is thirty degrees and if we look at the graph you can see that that's the solution here, the one between zero and ninety degrees. So our calculator gives one solution, how do we find this other solution? Well, we can use this symmetry property of the graph because you can see that this value, thirty degrees, well it’s thirty to the right of zero, so this value by the symmetry must be thirty to the left of a hundred and eighty, so we can find it that way.

So by the symmetry in the line theta equals ninety degrees, this line over here, well by that symmetry another solution is theta equals a hundred and eighty degrees minus thirty degrees, so that’s a hundred and fifty degrees. So let me show you that on the graph. Thirty degrees was thirty to the right of zero, so our other solution is thirty to the left of a hundred and eighty, and that’s a hundred and fifty degrees, there.

Well, if we ask where does this line y equals a half cross y equals sine theta it's only at those two points between nought and three hundred and sixty. So from the graph these two numbers, theta equals thirty degrees and theta equals a hundred and fifty degrees, are the only solutions between zero degrees and three hundred and sixty degrees, so we've answered that question.

Now let's look at this question about finding the solutions of cos theta equals minus a half between zero degrees and three hundred and sixty degrees.

So here we're asked to find all the solutions of the equation cos theta equals minus a half between zero degrees and three hundred and sixty degrees again, and we're going to use a similar approach. So let's start by drawing the graph of y equals cos theta this time. Now here's my ruler. My theta-axis, my y-axis, and I want to go up to three hundred and sixty degrees, and about halfway between I want a hundred and eighty degrees. Ninety degrees and two hundred and seventy degrees.

Now I need to know some standard values of the cosine function in order to - no, I'll start again.

Now I know some standard values of cos theta so let me plot those. First of all, cosine of zero degrees, well that's one. So there's one and a point's plotted. I'm going to need minus one in a moment so let me plot that. So cosine of zero degrees is one and cosine of ninety degrees is zero, so that's a point there. And then cosine of a hundred and eighty degrees, well this is minus one. Cosine of two hundred and seventy degrees, the graph's now going through that point, and cosine of three hundred and sixty degrees is back to one again, so the graph repeats itself after every three hundred and sixty degrees. So there's five points, and I've got to draw a smooth curve through those, so here I go. So the graph begins by falling from hyperbole down to zero, and then smoothly through minus one, and then back up again. Of course it carries on in both directions infinitely far. So there's my graph of y equals cos theta.

Well again it's not a very neat graph but let me point out a symmetry property that I'm going to use that the graph is symmetric under a reflection in this vertical line. So this line is here, goes through a hundred and eighty degrees. So that's the line theta equals a hundred and eighty degrees.

Okay, we're asked to solve this equation cosine of theta equals minus a half, so I'm going to draw a line at height minus a half. So there's minus a half. I use the ruler for this. So there's my line at height minus a half, well more or less minus a half. Let me label it y equals minus a half. And the solutions to this equation occur at the points where this straight line crosses the graph y equals cos theta. And you can see that there are two points here, let me mark those. This one and this one.

So they'll give me two values of theta on the theta axis, one here and one here. So I've got to find those values of theta. And I can find one of them by using the cos to the minus one function. So one solution of cos theta equals minus a half is theta equals cos to the minus one, so that's using the cos to the minus one button on your calculator, of minus a half. And if you

use your calculator to do that you'll find that the answer is a hundred and twenty degrees. So that's thirty degrees to the right of ninety in this case.

Well, how do we find this other solution? Well, we can use this symmetry property I talked about under reflection in the line theta equals a hundred and eighty degrees. So our solution a hundred and twenty degrees is thirty degrees to the right of ninety degrees, so this other solution must be thirty degrees to the left of two hundred and seventy degrees.

So by the symmetry in the line theta equals a hundred and eighty degrees, another solution is theta equals two hundred and seventy degrees minus thirty degrees, and that's equal to two hundred and forty degrees. So this, these two values are a hundred and twenty degrees and two hundred and forty degrees; thirty to the right of ninety and thirty to the left of two hundred and seventy.

Well, you can see from the graph that if you're only considering solutions that lie between zero degrees and three hundred and sixty degrees these are the only two possibilities. You'll find there are other possibilities to the right and the left but in this interval between zero and three hundred and sixty these are the only two. So from the graph, theta equals a hundred and twenty degrees and theta equals two hundred and forty degrees are the only solutions of this equation between zero degrees and three hundred and sixty degrees. So that's done.